06

In the previous chapter, we have seen that a current carrying conductor when kept in a magnetic field, experiences force and torque. Now, in this chapter, we will study that the reverse phenomenon is also possible, i.e. if we rotate the conductor (coil) in a magnetic field, then the current will flow in it.

ELECTROMAGNETIC INDUCTION

Current can be induced in the coils when these coils are rotated in a magnetic field. This has led to the alternate ways of generating current. When electromagnetic induction was discovered, only source of emf available were those of chemical nature such as dry cells, but at present large-scale production and distribution of energy became possible because of this phenomenon of Electromagnetic Induction (EMI). Faraday and Henry independently discovered the principle of magnetically induced emfs and found methods to convert mechanical energy into electrical energy. EMI formed the principle of two important electrical devices namely, electric generator and transformer, which has revolutionised the life styles of mankind.

CHAPTER CHECKLIST

- Faraday's Laws and Motional Electromotive Force
- Self and Mutual Induction

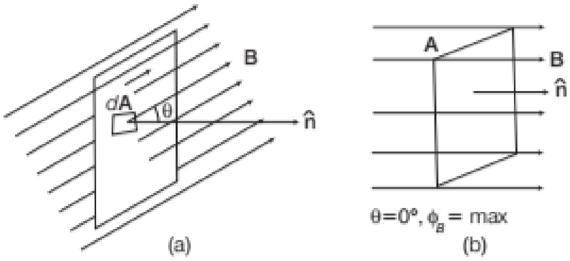
|TOPIC 1|

Faraday's Laws and Motional Electromotive Force

The phenomenon of generation of current or emf by changing the magnetic field is known as **Electromagnetic Induction** (EMI). The emf developed in the conductor by the process of EMI is known as **induced emf** and if the conductor is in the form of a closed loop, then the current flowing through the conductor is known as **induced current**. It is the reverse process of magnetic field production by electric current. The phenomenon of EMI was discovered by Michael Faraday in 1831, which is not merely of theoretical or academic interest but also of practical utility. We cannot imagine a world with no electricity, no electric lights, no trains, no telephones, no personal computers. Hence, today's civilisation owes a great deal to the discovery of EMI.

MAGNETIC FLUX

The total number of magnetic field lines crossing through any surface normally, when it is placed in a magnetic field is known as the magnetic flux of that surface.



(a) (b)

Suppose a loop enclosing an area **A** is placed in a uniform magnetic field **B**. Then, the magnetic flux through the loop is given by

$$\phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

When the magnetic field is perpendicular to that plane of the loop, then magnetic flux will be

$$\phi_B = BA = \text{maximum value} \quad \text{...(i)}$$

This means that $B = \frac{\phi_B}{A}$, i.e. magnetic field strength B is the magnetic flux per unit area and is called **magnetic flux density** or **magnetic induction**.

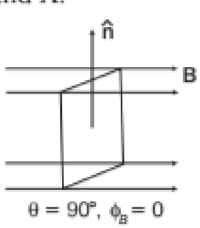
When the magnetic field B is not perpendicular to area A rather it is inclined at an angle θ with respect to the normal to the surface.

The magnetic flux becomes

$$\phi_B = \mathbf{B} \cdot \mathbf{A} = |\mathbf{B}||\mathbf{A}| \cos\theta = BA\cos\theta \quad \text{...(ii)}$$

where, θ is the smaller angle between **B** and **A**.

If a plane is parallel to the magnetic field, then no field line will pass through it and the magnetic flux linked with that plane will be zero.



From Eq.(ii), it is clear that the flux can be varied by changing anyone or more of the terms **B**, **A** and θ.

The flux can also be altered by changing the shape of the coil (by stretching or by compressing) in a magnetic field or rotating a coil in a magnetic field, such that the angle θ between **B** and **A** changes.

The SI unit of magnetic flux (ϕ_B) is tesla-metre square, which is also called weber (abbreviated Wb).

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T-m}^2$$

1 weber is the amount of magnetic flux over an area of 1m^2 held normal to a uniform magnetic field of 1 tesla (T).

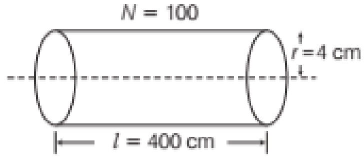
The CGS unit of magnetic flux (ϕ_B) is maxwell (Mx).

where, $1 \text{ weber} = 10^8 \text{maxwell}$

Magnetic flux is a scalar quantity and its dimensional formula is $[ML^2T^{-2}A^{-1}]$.

EXAMPLE |1| A long solenoid of radius 4 cm, length 400 cm carries a current of 3 A. The total number of turns is 100. Assuming ideal solenoid, find the flux passing through a circular surface having centre on axis of solenoid of radius 3 cm and is perpendicular to the axis of solenoid (i) inside and (ii) at the end of solenoid.

Sol.



Number of turns per unit length is given by

$$n = \frac{N}{l} = \frac{100}{4} = 25 \text{ turns/m}$$

(i) Magnetic field of a solenoid at a point inside is

$$B = \mu_0 ni$$

Area of cross-section of the solenoid, $A = \pi r_1^2$ and $\theta = 0°$

Magnetic flux,

$$\phi_B = BA\cos\theta$$
$$= \mu_0 ni\, \pi r_1^2 \cos 0°$$
$$= 4\pi \times 10^{-7} \times 25 \times 3 \times \pi \times (3 \times 10^{-2})^2$$
$$= 0.27 \times 10^{-6} \text{ Wb}$$
$$= 0.27\mu \text{ Wb}$$

(ii) At the end, magnetic field of solenoid is

$$B = \frac{1}{2}\mu_0 ni$$

$$\therefore \quad \phi_B = \frac{0.27}{2} = 0.135\mu \text{ Wb}$$

EXPERIMENTS OF FARADAY AND HENRY

The discovery and understanding of electromagnetic induction are based on a long series of experiments carried out by Faraday and Henry.

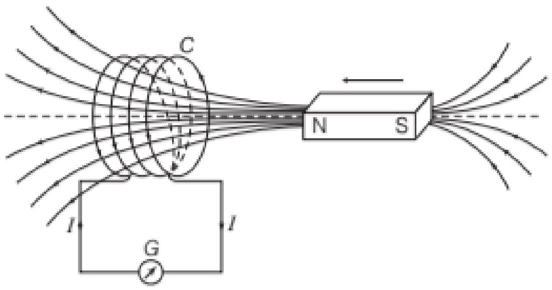
It is the relative motion between the magnet and closed coil, which is responsible for generation or induction of electric current in the coil.

Whenever magnetic field linked with a closed coil changes, an emf is induced in the coil, which is called **induced emf**.

First Experiment
(Current Induced by a Magnet)

Consider a coil C of few turns of conducting material insulated from one another and is connected to a sensitive galvanometer G.

Whenever there is a relative motion between the coil and magnet, the galvanometer shows a deflection indicating that current is induced in the coil.

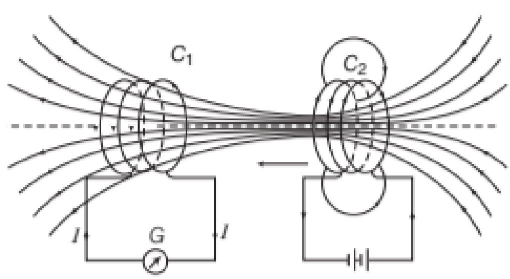


EMI with a stationary coil and moving magnet

Therefore, relative motion between the magnet and the coil generates electric current in the coil. So, the current generated is called **induced current**.

Second Experiment
(Current Induced by a Current)

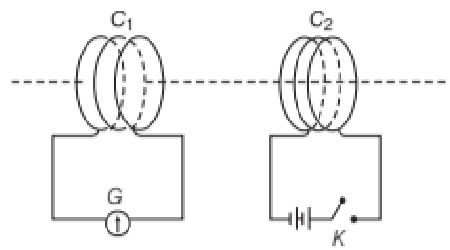
When the bar magnet is replaced by a second coil C_2 connected to a battery, the steady current in coil C_2 produces a steady magnetic field. As coil C_2 is moved towards coil C_1, the galvanometer shows a deflection. This indicates that electric current is induced in coil C_1.



EMI with one coil stationary and another moving

When coil C_2 is moved away, the galvanometer shows a deflection again but this time, in the opposite direction. The deflection lasts as long as coil C_2 is in the motion.

Third Experiment
(Current Induced by Changing Current)



EMI with changing current in one coil

The figure shows two coils C_1 and C_2 held stationary. Coil C_1 is connected to galvanometer G, while the second coil C_2 is connected to a battery through a tapping key K.

It is observed that the galvanometer shows a momentary deflection when the tapping key K is pressed.If the key is pressed continuously, there is no deflection in the galvanometer. When the key is released, a momentary deflection is observed again but in opposite direction. All experimental observations lead us to conclude that induced emf appears in a coil, whenever the amount of magnetic flux linked with the coil changes.

Note Presence of magnetic flux is not enough. The amount of magnetic flux linked with the coil must be change in order to produce an induced emf in the coil.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

The two laws of electromagnetic induction given by Faraday are stated below

Faraday's First Law

Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in it. The SI unit of this induced emf is volt (V).The actual number of magnetic field lines passing through the circuit does not depend on the values of the induced emf. Induced current is determined by the rate at which the magnetic flux changes.

Faraday's Second Law

The magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through the circuit. Mathematically, Faraday's second law can be expressed as,

Induced emf $\propto$ Rate of change of magnetic flux i.e. $$e = \frac{-d\phi_B}{dt}$$

$$\left[\because \text{rate of change of magnetic flux} = \frac{\phi_2 - \phi_1}{t_2 - t_1}\right]$$

The negative sign in above relation indicates that the induced emf in the loop due to changing flux always opposes the change in the magnetic flux. In other words, the direction of induced emf is such that it always opposes the change in magnetic flux linked with the circuit. In the case of a closely wound coil of N turns, the change of flux associated with each turn is same. Therefore, the expression for the total induced emf is given by

$$e = -N\frac{d\phi_B}{dt}$$

The induced emf can be increased by increasing the number of turns N of a closed coil.

Induced Emf and Current

If N is the number of turns and R is the resistance of a coil, and the magnetic flux linked with its each turn changes by $d\phi$ in short time interval dt, then

Induced emf in the coil, $e = -N\frac{d\phi_B}{dt}$

Induced current flowing through the coil,

$$I = \frac{e}{R} = -\frac{N}{R}\cdot\frac{d\phi_B}{dt} \qquad \left[\because I = \frac{V}{R}\right]$$

Electric charge flows due to induced current,

$$q = Idt = \frac{N}{R}d\phi_B$$

EXAMPLE |2| A magnetic field of flux density 10 T acts normally to the coil of 50 turns having 100 cm^2 area. Find emf induced, if the coil is removed from the magnetic field in 0.15 s.

Sol. Given, $B = 10\text{ T}$, $N = 50$ turns,

$A = 100\text{ cm}^2 = 10^{-2}\text{ m}^2$, $dt = 0.15\text{ s}$

Magnetic flux linked with the coil initially,

$\phi_1 = NBA = 50 \times 10 \times 10^{-2} = 5\text{ Wb}$

But magnetic flux linked with the coil finally, i.e.

(when removed from the magnetic field), $\phi_2 = 0$.

$\therefore$ Emf induced, $e = \frac{-d\phi}{dt} = -\left(\frac{\phi_2 - \phi_1}{dt}\right)$

$= -\left(\frac{0-5}{0.15}\right)$

$= 33.33\text{ V}$

EXAMPLE |3| A square loop of side 10 cm and resistance 0.5 Ω is placed vertically in the East-West plane. A uniform magnetic field of 0.10 T is set up across the plane in the North-East direction. The magnetic field is decreased to zero in 0.70 s at a steady rate. Determine the magnitudes of induced emf and current during this time interval.

Sol. Given, $B = 0.10\text{ T}$, $A = 100\text{ cm}^2 = 10^{-2}\text{ m}^2$, $\theta = 45°$

(as the angle made by area vector of the loop with magnetic field is 45°),

$R = 0.5\,\Omega$, $dt = 0.70\text{ s}$

Initial magnetic flux, $\phi_1 = BA\cos\theta$

$= 0.10 \times 10^{-2} \times \cos 45°$ $\left(\because \cos 45° = \frac{1}{\sqrt{2}}\right)$

$= \frac{10^{-3}}{\sqrt{2}}\text{ Wb}$

As, final magnetic flux, $\phi_2 = 0$.

$\therefore$ Magnitude of induced emf is

$$e = -\frac{d\phi}{dt} = -\left(\frac{\phi_2 - \phi_1}{dt}\right) = -\left(\frac{0 - \frac{10^{-3}}{\sqrt{2}}}{0.70}\right)$$

$= 10^{-3}\text{ V} = 1.0\text{ mV}$

$\therefore$ Magnitude of induced current,

$I = \frac{e}{R} = \frac{10^{-3}\text{ V}}{0.5\,\Omega}$

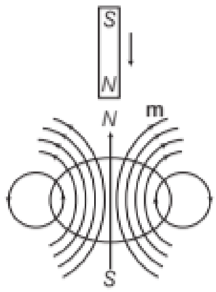
$= 2 \times 10^{-3}\text{ A}$

$= 2\text{ mA}$

LENZ'S LAW

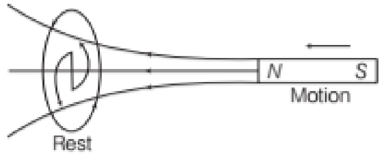
According to this law, the polarity of emf induced is such that, it tends to produce a current which opposes the change in magnetic flux that produced it.

Illustration of Lenz's Law

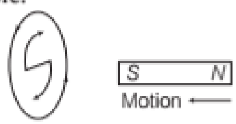


As the magnet is moved towards the loop, a particular amount of current is induced in the loop. The magnetic field is produced by the current, with magnetic dipole moment **m** oriented, so as to oppose the motion of magnet. Thus, the induced current must be counterclockwise as shown in the figure. When the North pole of a magnet moves towards a stationary loop, an induced current I flows in anti-clockwise sense as seen from the above, at which the magnet is located.

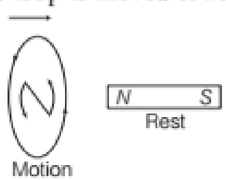
The anti-clockwise sense corresponds to the generation of North pole which opposes the motion of the approaching N-pole of the magnet.

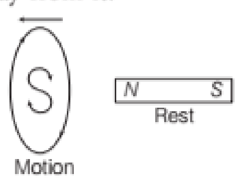


When the North pole of the magnet is moved away from the loop, the current I flows in the clockwise sense, which corresponds to the generation of South pole as shown in figure. The induced South pole opposes the motion of the receding North pole.



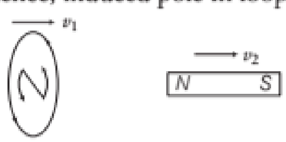
The directions of current induced in all above cases remain same, if instead of the loop, the magnet is kept stationary and loop is moved towards or away from it.



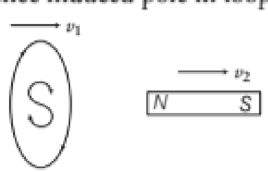


If both the loop and magnet are in relative motion w.r.t. each other, the induced pole on the loop facing magnet is according to Lenz's law.

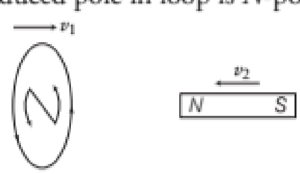
(i) When $v_1 > v_2$, i.e. the loop is approaching towards N-pole, hence, induced pole in loop is N-pole.



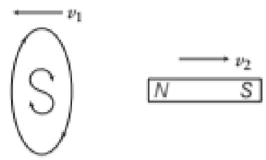
(ii) When $v_1 < v_2$, i.e. the loop is receding away from N-pole, hence induced pole in loop is S-pole.



(iii) When loop and magnet having opposite directions of velocities, then loop is approaching towards N-pole, hence, induced pole in loop is N-pole.



(iv) When loop and magnet having opposite directions of velocities and loop is receding away from N-pole, then induced pole in loop is S-pole.

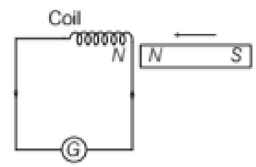


Thus, Lenz's law is used to find the direction of induced current in a closed circuit.

Note This topic has been frequently asked in previous years 2017, 2014, 2013, 2012, 2011, 2010.

Lenz's Law and Conservation of Energy

Lenz's law is in accordance with the law of conservation of energy.

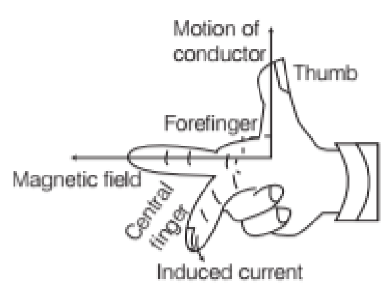


In the alongside circuit, when N-pole of magnet is moved towards the coil, the front face of the coil acquires North polarity. Thus, work has to be done against the force of repulsion in bringing the magnet closer to the coil.

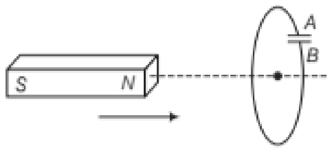
When N-pole of magnet is moved away, South pole develops on the front face of the coil. Therefore, work has to be done against the force of attraction in taking the magnet away from the coil. This mechanical work in moving the magnet w.r.t. the coil changes into electrical energy producing induced current. Hence, energy transformation takes place.When we do not move the magnet, work done is zero. Therefore, induced current is also zero.Hence, Lenz's law obeys the law of conservation of energy.

Fleming's Right Hand Rule

If we stretch the thumb, the forefinger and the central finger of right hand in such a way that all these three are mutually perpendicular to each other and if thumb represents the direction of motion of the conductor and the forefinger represents the direction of magnetic field, then central finger will represent the direction of induced current as shown below.



EXAMPLE |4| In the given figure, a bar magnet is quickly moved towards a conducting loop having a capacitor. Predict the polarity of the plates A and B of the capacitor.



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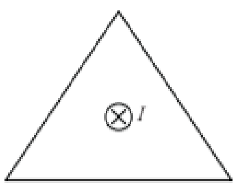
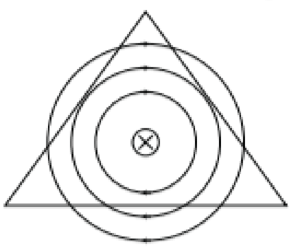
> **Hints:** As, the magnet moves towards the coil, flux linked with the coil, increases, hence according to the Lenz's law, it will oppose the change.

Sol. Here the North pole is approaching towards the magnet, so the induced current in the face of loop viewed from left side will flow in such a way that it will behave like North pole or South pole is developed in loop when viewed from right hand side of the loop. The flow of induced current is clockwise hence, A acquires positive polarity and B acquires negative polarity.

EXAMPLE |5| A current carrying straight wire passes inside a triangular coil as shown in figure. The current in the wire is perpendicular to paper inwards. Find the direction of the induced current in the loop, if current in the wire is increased.

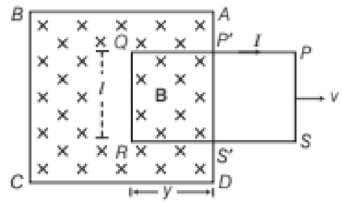


Sol. Magnetic field lines around the current carrying wire are as shown in figure below. Since, the lines are tangential to the loop ($\theta = 90°$), the flux passing through the loop is zero, whether the current is increased or decreased. Hence, change in flux is zero. Therefore, induced current in the loop will be zero.



MOTIONAL ELECTROMOTIVE FORCE AND FARADAY'S LAW

Consider a uniform magnetic field **B** confined to the region $ABCD$ and a coil $PQRS$ is placed inside the magnetic field. At any time t, the part $P'Q = S'R = y$ of the coil is inside the magnetic field. Let l be the length of the arm of the coil.



Inducing current by changing the area of the rectangular loop

Area of the coil inside the magnetic field at time t,

$$\Delta S = QR \times RS' = ly$$

Magnetic flux linked with the coil at any time t,

$$\phi = B\Delta S = Bly$$

The rate of change of magnetic flux linked with the coil is given by

$$\frac{d\phi}{dt} = \frac{d}{dt}(Bly)$$

$$= Bl\frac{dy}{dt} = Blv \qquad \left[\because \frac{dy}{dt} = v\right]$$

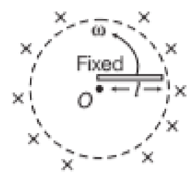
where, v is the velocity with which the coil is pulled out of the magnetic field.

If e is the induced emf, then according to Faraday's law,

$$e = -\frac{d\phi}{dt} \text{ or } \boxed{e = -Blv}$$

From Fleming's right hand rule, the current due to induced emf will flow from the end R to Q, i.e. along $SRQP$ in the coil. This induced electromotive force (emf) Blv is called **motional emf**.

Note When a conducting rod of length l fixed at its one end moves on a circular path with angular velocity ω in a uniform magnetic field B normal to it, then induced emf produced in it is $e = \frac{1}{2}B\omega l^2$.



EXAMPLE |6| A wire of length 0.3 m moves with a speed of 20 m/s perpendicular to the magnetic field of induction 1 Wb/m^2. Calculate the induced emf.

Sol. Given, velocity, $v = 20$ m/s

Length, $l = 0.3$ m

Angle, $\theta = 90°$

Magnetic field, $B = 1\ Wb/m^2$

As, induced emf, $e = Blv$

$\therefore \quad e = 1 \times 0.3 \times 20 = 6V$

EXAMPLE |7| A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the earth's magnetic field H_E at a place. If $H_E = 0.4$ gauss at the place, what is the induced emf between the axle and the rim of the wheel? Take, 1 gauss $= 10^{-4}$ T.

Sol. Induced emf $= \frac{1}{2} B\omega l^2 = (1/2)\,\omega B R^2$

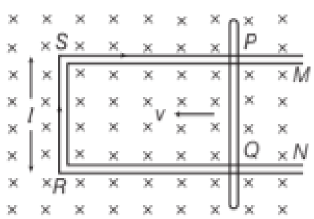
$$\left[\begin{array}{l} \because \omega = 2\pi f = 2\pi \frac{120}{60} = 4\pi \\ \text{and } l = 2R \end{array}\right]$$

$$= (1/2) \times 4\pi \times 0.4 \times 10^{-4} \times (0.5)^2$$
$$= 6.28 \times 10^{-5} \text{ V}$$

The number of spokes is immaterial because the emf's across the spokes are in parallel.

ENERGY CONSIDERATION
(A QUANTITATIVE STUDY)

Let R be the resistance of movable arm PQ of the irregular conductor. We assume that the remaining arms QR, RS and SP have negligible resistances compared to R.



Thus, overall resistance of the rectangular loop is R and this does not change as PQ is moved.

Current I in the loop is given by

$$I = \frac{e}{R} = \frac{Blv}{R} \qquad \ldots\text{(i)}$$

Due to the presence of the magnetic field, there is a force on the arm PQ. This force is directed outwards in the direction opposite to the velocity of the rod.

The magnitude of this force is given by magnetic force

i.e. $\quad F = IlB$ or $\boxed{F = \frac{B^2 l^2 v}{R}}$

Alternatively, the arm PQ is being pushed with a constant speed v. Power required to do this is given by

$$P = Fv \text{ or } \boxed{P = \frac{B^2 l^2 v^2}{R}} \qquad \ldots\text{(ii)}$$

The agent that does this work is mechanical energy.

This mechanical energy is dissipated as joule heat and is given by

$$P_J = I^2 R \text{ or } P_J = \left(\frac{Blv}{R}\right)^2 R \text{ or } \boxed{P_J = \frac{B^2 l^2 v^2}{R}}$$

This is identical to Eq.(ii).

Thus, mechanical energy, which was required to move the arm PQ is converted into electrical energy and then to thermal energy.

Note The magnetic flux linked with a loop does not change when

(i) magnet and loop are moving with the same velocity.

(ii) magnet is rotated around its axis without changing its distance from the loop.

(iii) loop is moved in a uniform magnetic field and the whole of the loop remains in the field.

Induced Quantities and Their Formulae

S.No.	Name of the quantity	Formula	Circuit open/ closed	Dependence upon resistance	SI unit
1.	Induced emf	$e = -\frac{d\phi}{dt}$	Open or closed	No	volt
2.	Induced current	$I = -\frac{d\phi}{Rdt}$	Closed	Yes	ampere
3.	Charge flown due to induced current	$dq = -\frac{d\phi}{R}$	Closed	Yes	coulomb
4.	Power required to pull a loop out of a magnetic field	$P = \frac{B^2 l^2 v^2}{R}$	Open or closed	Yes	watt

Induced Current in a Circuit

If R is the electrical resistance of the circuit, then induced current in the circuit is given by

$$I = \frac{e}{R}$$

If induced current is produced in a coil rotated in a uniform magnetic field, then

$$I = \frac{NBA\omega \sin\omega t}{R} = I_0 \sin\omega t$$

where, $I_0 = \frac{NBA\omega}{R}$ = peak value of induced current

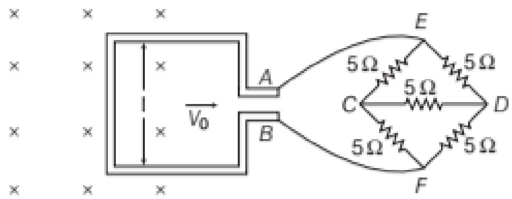
N = number of turns in the coil,

B = magnetic field,

ω = angular velocity of rotation

and $\quad A$ = area of cross-section of the coil.

EXAMPLE |8| A square metal wire loop of side 20 cm and resistance $2\,\Omega$ is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 1$ Wb/m^2 as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop. The loop is connected to a network of resistance each of value $5\,\Omega$. The resistances of the lead wires BF and AE are negligible. What should be the speed of the loop, so as to have a steady current of 2 mA in the loop? Give the direction of current in the loop.



Sol. From the figure, we see that, network *CEDF* is balanced Wheatstone bridge, so no current will flow in branch *CD*.

So, the equivalent resistance of *CEDF* network is

$$R_{eq} = \frac{10 \times 10}{10 + 10} = 5\Omega$$

Resistance of loop = 2 Ω

$$R_{total} = 2 + R_{eq} = 2 + 5 = 7\ \Omega$$

We know that, induced emf, $e = Bv_0 l$

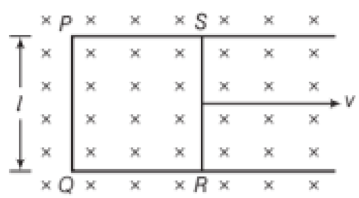
and induced current, $I = \frac{e}{R_{total}} = \frac{Bv_0 l}{7}$

$$\Rightarrow \quad 2 \times 10^{-3} = \frac{1 \times v_0 \times 0.2}{7}$$

$$\Rightarrow \quad v_0 = 7 \times 10^{-2}\ \text{m/s} = 7\ \text{cm/s}$$

As flux is decreasing, so induced current *I* will be clockwise.

EXAMPLE |9| Figure shows a rectangular conducting loop *PQRS* in which arm *RS* of length *l* is movable. The loop is kept in a uniform magnetic field *B* directed downward perpendicular to the plane of the loop. The arm *RS* is moved with a uniform speed *v*.



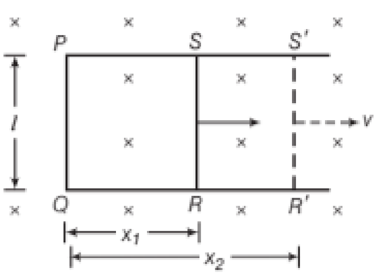
Deduce the expression for

(i) the emf induced across the arm *RS*

(ii) the external force required to move the arm and

(iii) the power dissipated as heat. **All India 2009**

Sol. According to the question,



(i) Let *RS* moves with speed *v* rightward and also *RS* is at distances x_1 and x_2 from *PQ* at instants t_1 at t_2, respectively.

$\therefore$ At t_1, flux linked with loop 1, i.e. *PQRS*, $\phi_1 = B(lx_1)$

Similarly, at instant t_2, flux linked with loop 2, i.e. *PQR′S′*, $\phi_2 = B(lx_2)$

$\therefore$ Change in flux, $\Delta\phi = \phi_2 - \phi_1 = Bl(x_2 - x_1) = Bl\Delta x$

$$\Rightarrow \quad \frac{\Delta\phi}{\Delta t} = Bl\frac{\Delta x}{\Delta t} = Blv \qquad \left[\because v = \frac{\Delta x}{\Delta t}\right]$$

By Faraday's law, magnitude of induced emf, $e = vBl$.

(ii) If resistance of loop is *R*, then $I = \frac{vBl}{R}$

$$\therefore \text{ Magnetic force} = IBl \sin 90° = \left(\frac{vBl}{R}\right)Bl$$

$$= \frac{vB^2l^2}{R} \qquad [\because \sin 90° = 1]$$

$\therefore$ External force must be equal to magnetic force and in opposite directions.

$$\therefore \text{ External force} = \frac{vB^2l^2}{R}$$

(iii) As, $P = I^2R = \left(\frac{vBl}{R}\right)^2 \times R = \frac{v^2B^2l^2}{R^2} \times R$

$$\therefore \quad P = \frac{v^2B^2l^2}{R}$$

EDDY CURRENTS

The currents induced in bulk pieces of conductors, when the magnetic flux linked with it changes, are known as eddy currents. These currents are always produced in a plane, perpendicular to the direction of magnetic field. They show both heating and magnetic effects.

The magnitude of eddy current is given by

$$I = \frac{\text{Induced emf}}{\text{Resistance}} = \frac{e}{R}$$

According to Faraday's law, $e = -\frac{d\phi}{dt}$, then $I = -\frac{d\phi/dt}{R}$

The direction of eddy currents can be given by Lenz's law or by Fleming's right hand rule. However, their flow patterns resemble swirling eddies in water. That is why, they are called **eddy currents**. These were discovered by **Foucault** in 1895 and hence, they are also named as **Foucault current**. e.g. When we move a metal plate out of a magnetic field, the relative motion of the field and the conductor again induces a current in the conductor. The conduction electrons build up the induced current whirl around within the plate as, if they were caught in an eddy of water. This is called the eddy current.

Undesirable Effects of Eddy Currents

Eddy currents are produced inside the iron cores of the rotating armatures of electric motors and dynamos and also in the cores of transformers, which experience flux changes, when they are in use. They cause unnecessary heating and wastage of power. The heat produced by eddy currents may even damage the insulation of coils. They are minimised by using laminations of metal to make a metal core. The laminations are separated by an insulating material. The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths. This arrangement reduces the strength of eddy currents.

Applications of Eddy Currents

Eddy currents are useful in many ways. Some of the important applications of eddy currents are as given below

(i) **Electromagnetic damping** In order to immediately bring the moving coil of a galvanometer to rest, we make the use of electromagnetic damping which uses eddy currents to bring the coil to rest. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest.

(ii) **Induction furnace** In this, high temperature can be produced by using eddy currents. We generally use induction furnace in preparation of alloys by melting the constituents of metal. A coil is wound over the metal which needs to be melted and through the coil, we pass high frequency alternating current.

The eddy current generated in the metal produces high temperature to melt the metal.

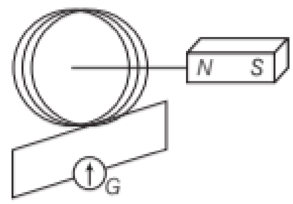
(iii) **Electric power meters** Old electric power meters (analog type) had a metallic disc. The disc rotates due to generation of eddy currents which are produced due to sinusoidally varying currents in the coil.

(iv) **Magnetic braking in electronic trains** Some electric powered trains make use of strong electromagnets which are situated above the rails. These electromagnets are used to produce eddy currents in the rails which oppose the motion of the train and thus stop it. In this case, as there is no mechanical linkage, the braking effect is smooth.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Current in the coil is larger



(a) when the magnet is pushed towards the coil faster
(b) when the magnet is pulled away the coil faster
(c) Both (a) and (b)
(d) Neither (a) nor (b)

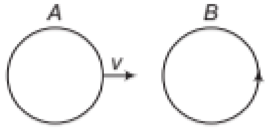
2. The instantaneous magnetic flux linked with a coil is given by $\phi = (5t^3 - 100t + 300)$ Wb. The emf induced in the coil at time $t = 2$ s is

(a) – 40 V (b) 40 V
(c) 140 V (d) 300 V

3. There are two coils A and B as shown in figure. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter clockwise. B is kept stationary when A moves. We can infer that

NCERT Exemplar

(a) there is a constant current in the clockwise direction in A
(b) there is a varying current in A
(c) there is no current in A
(d) there is a constant current in the counter clockwise direction in A



4. A horizontal straight wire 20 m long extending from east to west is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of the earth's magnetic field 0.30×10^{-4} Wbm^{-2}. The instantaneous value of the emf induced in the wire will be

(a) 6.0 mV (b) 3 mV
(c) 4.5 mV (d) 1.5 mV

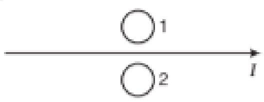
VERY SHORT ANSWER Type Questions

5. Two coils of wire A and B are placed mutually perpendicular. When a current induced is changed in any one coil, will the current induced in another coil?

6. A long straight current carrying wire passes normally through the centre of circular loop. If the current through the wire increases, will there be an induced emf in the loop? Justify.

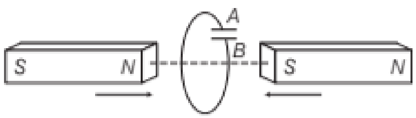
Delhi 2017

7. What is the direction of induced currents in metal rings 1 and 2, when current I in the wire is increasing steadily?

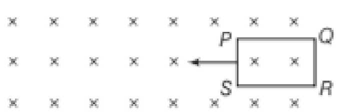


All India 2017

8. In the figure given, mark the polarity of plates A and B of a capacitor when the magnets are quickly moved towards the coil. **All India 2017 C**



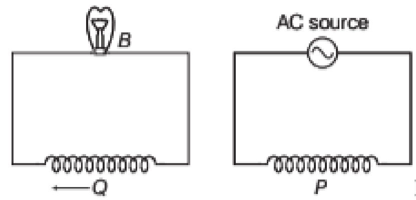
9. The closed loop $PQRS$ of wire is moved into a uniform magnetic field at right angles to the plane of the paper as shown in the figure. Predict the direction of the induced current in the loop. **Foreign 2012**



11. A coil Q is connected to low voltage bulb B and placed near another coil P as shown in the figure. Give reasons to explain the following observations

(i) The bulb B lights.

(ii) Bulb gets dimmer, if the coil Q is moved towards left.



Delhi 2010

12. Two identical loops, one of copper and the other of aluminium are rotated with the same angular speed in the same magnetic field. Compare

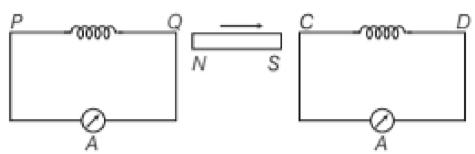
(i) the induced emf and

(ii) the current produced in the two coils. Justify your answer. **All India 2010**

13. State Lenz's law. A metallic rod held horizontally along East-West direction, is allowed to fall under gravity. Will there be an emf induced at its ends? Justify your answer.

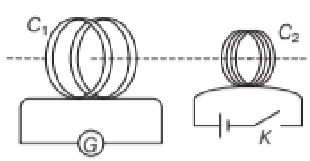
Delhi 2013

14. A bar magnet is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the directions of induced current in each coil. **All India 2012**



SHORT ANSWER Type Questions

10. A current is induced in coil C_1 due to the motion of current carrying coil C_2.



(i) Write any two ways by which a large deflection can be obtained in the galvanometer G.

(ii) Suggest an alternative device to demonstrate the induced current in place of a galvanometer. **Delhi 2011**

15. A rectangular loop of length l and breadth b is placed at distance of x from infinitely long wire carrying current i such that the direction of current is parallel to breadth. If the loop moves away from the current wire in a direction perpendicular to it with a velocity v, what will be the magnitude of emf in the loop?

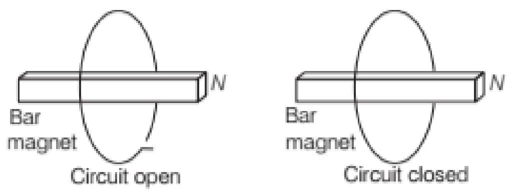
16. A metallic rod of length L is rotated with angular frequency of ω with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius L, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and a uniform magnetic field B parallel to the axis is present everywhere. Deduce the expression for the emf between the centre and the metallic ring.

Delhi 2012

17. Why is the coil of dead beat galvanometer wound on a metal frame?

18. Consider a magnet surrounded by a wire with an ON/OFF switch as shown in the figure. If the switch is thrown from the OFF position (open circuit) to the ON position (closed circuit), will a current flow in the circuit? Explain.

NCERT Exemplar



> **Hints :** The magnetic flux linked with a uniform surface area A in a uniform magnetic field is given by $\phi = \mathbf{B} \cdot \mathbf{A} = BA \cos\theta$. So, flux linked will changes, only when either B or A or the angle between B and A changes.

19. A wire in the form of tightly wound solenoid is connected to a DC source and carries a current I. If the coil is stretched, so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain.

NCERT Exemplar

> **Hints:** Here, the application of Lenz's law is tested through this problem.

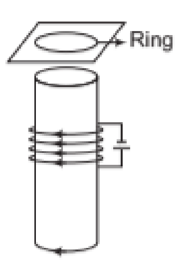
20. A solenoid is connected to a battery, so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease? Explain.

NCERT Exemplar

21. (i) A metal ring is held horizontally and bar magnet is dropped through the ring with its length along the axis of the ring. What will be the acceleration of a falling magnet?

(ii) Consider a metal ring kept on top of a fixed solenoid (say on a cardboard) (see figure). The centre of the ring coincides with the axis of the solenoid. If the current is suddenly switched ON, the metal ring jumps up. Explain.

NCERT Exemplar

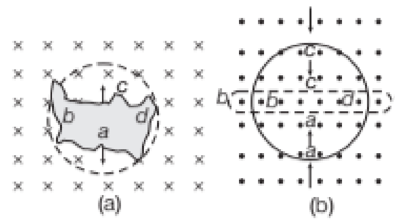


LONG ANSWER Type I Questions

22. Use Lenz's law to determine the direction of induced current in the situations described by figure.

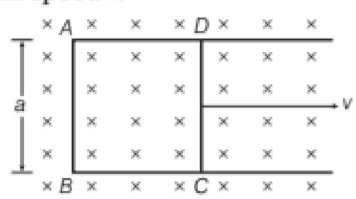
(i) A wire of irregular shape turning into a circular shape.

(ii) A circular loop being deformed into a narrow straight wire. **NCERT**



23. A metallic rod of length l is moved perpendicular to its length with velocity v in a magnetic field $\mathbf{B}$ acting perpendicular to the plane in which rod moves. Derive the expression for the induced emf. **All India 2017 C**

24. Figure shows a rectangular conducting loop $ABCD$ in which arm CD of length a is movable. The loop is kept in a uniform magnetic field B directed downward perpendicular to the plane of the loop. The arm CD is moved with a uniform speed v.



Deduce an expression for

(i) the emf induced across the arm CD

(ii) the external force required to move the arm and

(iii) the power dissipated as heat.

25. (i) A rod of length l is moved horizontally with a uniform velocity v in a direction perpendicular to its length through a region in which a uniform magnetic field is acting vertically downward. Derive the expression for the emf induced across the ends of the rod.

(ii) How does one understand this motional emf by invoking the Lorentz force acting on the free charge carriers of the conductor? Explain. **All India 2014**

26. A metallic rod of length l is rotated with a frequency ν with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius r, about an axis passing through the centre and perpendicular to the plane of the ring.
A constant uniform magnetic field B parallel to the axis is present everywhere. Using Lorentz force, explain how emf is induced between the centre and the metallic ring and hence obtain the expression for it? **Delhi 2013**

LONG ANSWER Type II Questions

27. A metallic rod of length l and resistance R is rotated with a frequency ν, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius l, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and a uniform magnetic field B parallel to the axis is present everywhere.

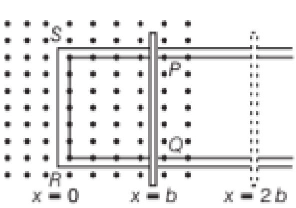
(i) Derive the expression for the induced emf and the current in the rod.

(ii) Due to the presence of the current in the rod and of the magnetic field, find the expression for the magnitude and direction of the force acting on this rod.

(iii) Hence, obtain the expression for the power required to rotate the rod. **All India 2014C**

28. State Faraday's law of electromagnetic induction. Figure shows a rectangular conductor $PQRS$ in which the conductor PQ is free to move in a uniform magnetic field B perpendicular to the plane of the paper. The field extends from $x = 0$ to $x = b$ and is zero for $x > b$. Assume that only the arm PQ possesses resistance r. When the arm PQ is pulled outward from $x = 0$ to $x = 2b$ and is then moved backward to $x = 0$ with constant speed v, obtain the expressions for the flux and the induced emf.

Sketch the variation of these quantities with distance $0 \le x \le 2b$. **All India 2010**



NUMERICAL PROBLEMS

29. A rectangular loop of area 20 cm × 30 cm is placed in magnetic field of 0.3 T with its plane

(i) normal to the field

(ii) inclined 30° to the field and

(iii) parallel to the field.

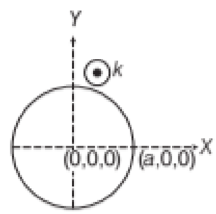
Find the flux linked with the coil in each case.

30. The magnetic flux through a coil perpendicular to the plane is given by $\phi = 5t^3 + 4t^2 + 2t$.
Calculate induced emf through the coil at $t = 2$s.

31. A circular coil of radius 10 cm, 500 turns and resistance 2 Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. Estimate the magnitude of the emf and current induced in the coil. Horizontal component of the earth's magnetic field at the place is 3×10^{-5} T. **NCERT Intext**

32. A magnetic field in a certain region is given by $\mathbf{B} = B_0 \cos(\omega t)\hat{\mathbf{k}}$ and a coil of radius a with resistance R, is placed in the xy-plane with its centre at the origin in the magnetic field as shown in the figure. Find the magnitude and the direction of the current at $(a, 0, 0)$ at

$t = \frac{\pi}{2\omega}, t = \frac{\pi}{\omega}$ and $t = \frac{3\pi}{2\omega}$.



NCERT Exemplar

33. A wheel with 15 metallic spokes each 60 cm long, is rotated at 360 rev/min in a plane normal to the horizontal component of the earth's magnetic field. The angle of dip at that place is 60°. If the emf induced between rim of the wheel and the axle is 400 mV, calculate the horizontal component of the earth's magnetic field at the place.
How will the induced emf change, if the number of spokes is increased?

All India 2017C

34. A horizontal straight wire 10 m long extending from East to West is falling with a speed of 5.0 m/s, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb/m^2.

(i) What is the instantaneous value of the emf induced in the wire?

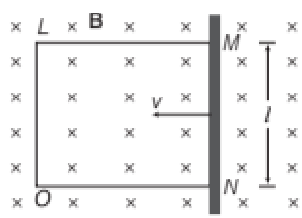
(ii) What is the direction of the emf?

(iii) Which end of the wire is at the higher electrical potential? **NCERT**

35. A jet plane is travelling towards West at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°? **NCERT**

36. A 1 m long conducting rod rotates with an angular frequency of 400 rad/s about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring. **NCERT**

37. A rectangular conductor *LMNO* is placed in a uniform magnetic field of 0.5 T. The field is directed perpendicular to the plane of the conductor.



When the arm *MN* of length 20 cm is moved towards left with a velocity of 10 ms^{-1}, calculate the emf induced in the arm. Given, the resistance of the arm to be 5 Ω (assuming that other arms are of negligible resistance), find the value of the current in the arm. **All India 2013**

38. A rectangular loop of sides 8 cm and 2 cm with a small cut is moving out of a region of a uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the voltage developed across the cut, if velocity of loop is 1 cms^{-1} in a direction normal to the

(i) longer side?

(ii) shorter side of the loop?

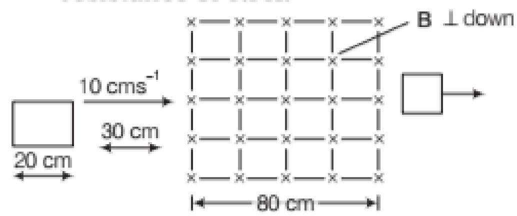
For how long does the induced voltage last in each case? **NCERT**

39. A square loop of side 20 cm is initially kept 30 cm away from a region of a uniform magnetic field of 0.1 T as shown in the figure. It is then moved towards the right with a velocity of 10 cm s^{-1} till it goes out of the field. Plot a graph showing the variation of

(i) magnetic flux (ϕ) through the loop with time (t).

(ii) induced emf (ε) in the loop with time t.

(iii) induced current in the loop, if it has resistance of 0.1 Ω.



40. A square loop of side 12 cm with its sides parallel to X and Y-axes is moved with a velocity of 8 cm/s in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither a uniform in space nor constant in time. It has a gradient of 10^{-3} T/cm along the negative x-direction (i.e. it increases by 10^{-3} T/cm as one moves in the negative x-direction) and it is decreasing in time at the rate of 10^{-3} T/s. Determine the direction and magnitude of the induced current in the loop, if its resistance is 4.50 mΩ. **NCERT**

41. A circular coil of radius 8.0 cm and 20 turns rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, then calculate the maximum value of current in the coil. Calculate the average power loss due to joule heating. Where does this power come from? **NCERT**

HINTS AND SOLUTIONS

1. (c) Current will be larger, when the magnet is pushed faster towards the coil, also current is large when magnet is pulled faster away but now it is in opposite direction.

2. (b) Given, $\phi = (5t^3 - 100t + 300)$, $t = 2$ s

Induced electromotive force,

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(5t^3 - 100t + 300)$$

$$e = -5 \times 3t^2 + 100 = -5 \times 3(2)^2 + 100$$

$$= -5 \times 12 + 100 = -60 + 100 = 40 \text{ V}$$

3. (d) When the A stops moving the current in B become zero, it possible only if the current in A is constant. If the current in A would be variable, there must be an induced emf (current) in B even if the A stops moving.

4. (b) Induced emf across the ends of wire

$$e = B_H lv = 0.30 \times 10^{-4} \times 20 \times 5 = 3 \text{ mV}$$

5. No, this is because the magnetic field due to the current in coil (A or B) will be parallel to the plane of the other coil (B or A). Hence, the magnetic flux linked with the other coil will be zero and so no current will be induced in it.

6. The flux created by straight current carrying wire is depicted in the figure.

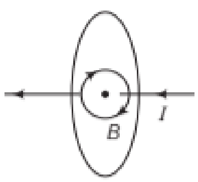
As, induced emf (e) $\propto$ rate of change of magnetic flux (ϕ_B)

and $\phi_B = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta$

Here, $\mathbf{B} \perp \mathbf{A} \Rightarrow \phi_B = BA\cos 90° = 0$

So, induced emf = 0

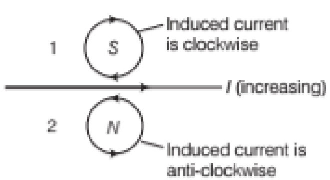
Hence, a change in current of wire will not create any emf in the loop.



7. Current in the wire is steadily increasing, so the induced current in rings 1 and 2 will flow in such a way that it opposes the increase of current.

So, it will flow in same direction. Now, from the figure, it is clear that the direction of induced current in

(i) ring 1 is clockwise, (ii) ring 2 is anti-clockwise.



8. From Lenz's law, the direction of induced current is in clockwise sense. This implies that plate A of the capacitor is at the higher potential than plate B, i.e. B will be negative plate, while A will be positive plate.

9. Since, magnetic flux increases, when the loop moves into a uniform magnetic field. So, the induced current should oppose this increase. Thus, the flow will be from $QPSRQ$, i.e. anti-clockwise.

10. (i) Large deflection in the galvanometer can be obtained when change in magnetic flux is fast. So, according to the diagram given in question,

(a) by moving quickly, the coil C_2 towards C_1 or by moving quickly the coil C_2 away from C_1.

(b) by switching off and on the key.

(ii) Alternating device in place of galvanometer can be LED or bulb.

11. (i) Due to varying current in P, the flux linked with P change and hence Q changes, which in turn induces the emf in Q and bulb B lights, where P and Q are coils.

(ii) When Q is moved left or it goes away from P, the lesser flux charge takes place in Q. This leads to decrease the value of rate of change of magnetic flux and hence, lesser emf and bulb B gets dimmer.

12. (i) The induced emf in both the loops will be same as areas of the loop and time periods are same as they are identical and rotated with same angular speed.

(ii) The current induces in Cu coil is more than Al coil as Cu coil has lesser resistance and $I \propto \frac{1}{R}$ (for the same voltage).

13. Lenz's law states that the polarity of induced emf is such that, it tends to produce a current which opposes the change in magnetic flux that produced it.

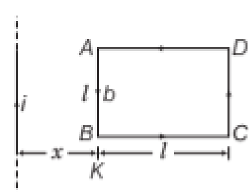
Yes, emf will be induced in the rod as there is change in magnetic flux.

14. From the figure, it is clear that North pole of the magnet is moving away from coil PQ, so the direction of current at end Q will flow in such a way that it will oppose the away moment of North pole, so it has to act as South pole. Hence, the direction of current will be clockwise.

Again, the South pole is approaching towards coil CD, so end C of the coil will act as South pole (to oppose the approaching South pole). Hence, the direction of current will be clockwise.

When a metallic rod held horizontally along East-West direction, is allowed to fall freely under gravity, i.e. fall from North to South direction, the intensity of magnetic lines of the earth's magnetic field changes through it, i.e. the magnetic flux changes and hence the induced emf in it. When we increase the number of turns, the induced emf will increase because induced emf is directly proportional to the number of turns.

15. Since, loop is moving away from the wire, so the direction of current in the loop will be as shown in the figure.



Net magnetic field on the loop due to wire,

$$B = \frac{\mu_0 i}{2\pi}\left[\frac{1}{x} - \frac{1}{l + x}\right] = \frac{\mu_0 il}{2\pi x\,(l + x)}$$

So, the magnitude of the emf in the loop,

$$e = vBb = \frac{\mu_0 il\,v\,b}{2\pi\;x\,(l + x)}$$

16. Angular velocity of rod, $\omega = \frac{2\pi}{T}$, where T = time period

$\therefore$ Charge in flux in one revolution $= BA = B(\pi L^2)$

According to Faraday's law of EMI, magnitude of induced emf,

$$e = \frac{\Delta\phi}{\Delta T} = \frac{B\pi L^2}{T} = \frac{B\pi L^2}{\left(\frac{2\pi}{\omega}\right)} \qquad \left[\because T = \frac{2\pi}{\omega}\right]$$

$$\therefore \qquad = \frac{1}{2}B\omega L^2$$

which is the required expression.

17. On switching ON, the current in a galvanometer, the coil of the galvanometer does not come to rest immediately. It oscillates about its equilibrium position but the coil of a dead beat galvanometer comes to rest immediately. It is due to the reason that the eddy currents are set up in the metallic frame, over which the coil is wound and the eddy currents oppose the oscillatory motion of the coil.

18. When the switch is thrown from the OFF position (open circuit) to the ON position (closed circuit), then neither B nor A and the angle between B and A does not change. Thus, no change in magnetic flux linked with coil occur, hence no electromotive force is produced and consequently, no current will flow in the circuit.

19. When the coil is stretched, so that there are gaps between successive elements of the spiral coil, i.e. the wires are pulled apart which lead to the flux leakage through the gaps. According to Lenz's law, the emf produced must oppose this decrease, which can be done by an increase in current. So, the current will increase.

20. When the iron core is inserted in the current carrying solenoid, the magnetic field increases due to the magnetisation of iron core and consequently, the flux increases. According to Lenz's law, the emf produced must oppose this increase in flux, which can be done by making decrease in current. So, the current will decrease.

21. (i) As the magnet falls, the magnetic flux linked with the ring increases. This induces emf in the ring which opposes the motion of the falling magnet, hence $a < g$.

(ii) When current is suddenly switched ON, magnetic flux linked with the solenoid and thus, with metal ring increases. Current is induced in the ring in anti-clockwise direction (as seen from top of the ring). Since, the direction of flow of current in the ring is opposite to the current in the solenoid, therefore they will repel each other and the ring jumps up.

22. (i) Here, the direction of magnetic field is perpendicularly inwards to the plane of paper. If a wire of irregular shape turns into a circular shape then its area increases (therefore the circular loop has greater area than the loop of irregular shape), so that the magnetic flux linked also increases. Now, the induced current is produced in a direction such that it decreases the magnetic field, i.e. the current will flow in such a direction, so that the wire forming the loop is pulled inward in all directions (to decrease the area), i.e. current is in anti-clockwise direction, i.e. along *adcba*.

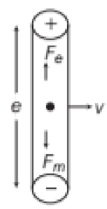
(ii) When a circular loop deforms into a narrow straight wire, the magnetic flux linked with it also decreases. The current induced due to change in flux will flow in such a direction that it will oppose the decrease in magnetic flux, so it will flow anti-clockwise, i.e. along *adcba* due to which the magnetic field produced will be out of the plane of paper.

23. Refer to text on page 255.

24. Refer to text Example 9 on page 257.

25. (i) Refer to text on page 255.

(ii) During motion, free electrons are shifted at one end due to magnetic force. So, due to polarisation of rod electric field is produced which applies electric force on free electrons in an opposite direction.



At equilibrium of Lorentz force,

$$F_e + F_m = 0$$

where, F_e = force due to electric field $= qE$

F_m = force due to magnetic field $= q(v \times B)$

$$\therefore \quad q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$$

$$\Rightarrow \quad \mathbf{E} = -\mathbf{v} \times \mathbf{B} = \mathbf{B} \times \mathbf{v}$$

$$\Rightarrow \quad |E| = Bv\sin\theta$$

***Case* I** If B, E and v are collinear, then charged particle is moving parallel or anti-parallel.

***Case* II** If v, E and B are mutually perpendicular, i.e. $\theta = 90°$, then Lorentz force is zero which means particle will pass through the field without any change.

26. Suppose the length of the rod is greater than the radius of the circle and rod rotates anti-clockwise. Suppose the direction of the rod at any instant be along positive y-direction and the direction of the magnetic field is along positive z-direction.

Then, using Lorentz law, we get the following,

$$\mathbf{F} = -e\,(\mathbf{v} \times \mathbf{B}) \Rightarrow \mathbf{F} = -e\,(v\,\hat{\mathbf{j}} \times B\,\hat{\mathbf{k}})$$

$$\Rightarrow \quad \mathbf{F} = -evB\hat{\mathbf{i}} \qquad [\because \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}]$$

Thus, the direction of force on the electrons is along X-axis. Thus, the electrons will move towards the centre, i.e. the fixed end of the rod. This movement of electrons will result in current having the direction opposite that of electrons and hence, it will produce emf in the rod between the fixed end and the point touching the ring. Let θ be the angle between the rod and radius of the circle at any time t.

Then, area swept by the rod inside the circle $= \frac{1}{2}\pi r^2\theta$

Now, induced emf $= B \times \frac{d}{dt}\left(\frac{1}{2}\pi r^2\theta\right) = \frac{1}{2}\pi r^2 B\frac{d\theta}{dt}$

$$= \frac{1}{2}\pi r^2 B\omega = \frac{1}{2}\pi r^2 B(2\pi\nu) = \pi^2 r^2 B\nu$$

27. (i) Refer to text on pages 255 and 256.
(ii) Refer to text on page 256.
(iii) Refer to text on page 256.

28. **For statement of Faraday's law of electromagnetic induction.** Refer to text on pages 252 and 253.

According to the given figure.

***Case* I** When PQ moves forward.

(i) For $0 \le x < b$

Magnetic field B exists in the region.

$\therefore$ Area of loop $PQRS = lx$

$\therefore$ Magnetic flux linked with loop $PQRS$,

$$\phi = BA = Blx$$

$\Rightarrow$ $\phi = Blx$ ...(i)

(ii) For $2b \ge x \ge b$ $(b > x \ge 0)$

$B = 0$

$\therefore$ Flux linked with loop $PQRS$ is a uniform and given by

$\phi' = Blb$ ...(ii)

$\Rightarrow$ $x = b$

Forward journey

Thus, for $b > x \ge 0$

Flux, $\phi = Blx \Rightarrow \phi \propto x$

For $2b \ge x \ge b$

Flux, $\phi = Bbl$ [constant]

Backward journey

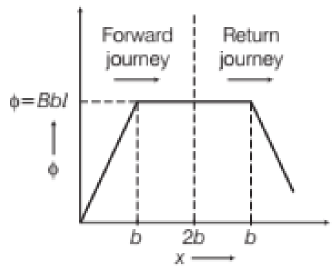
For $b \le x \le 2b$,

$\phi =$ constant $= Bbl$

For $0 \le x \le b$,

$\phi = Blx$ [decreasing]

Graphical representation



***Case* II** For $b > x \ge 0$, $B = 0$

As, $\phi = Blx$

$\Rightarrow$ $\frac{d\phi}{dt} = Bl\frac{dx}{dt} = B\nu l$ $\left[\because \nu = \frac{dx}{dt}\right]$

Induced emf, $e = -\frac{d\phi}{dt} = -\nu Bl$

For $2b \ge x \ge b$,

As, $\phi' = Bbl \Rightarrow \frac{d\phi'}{dt} = 0 \Rightarrow e = 0$

Forward journey

For $b > x \ge 0 \Rightarrow e = -\nu Bl$

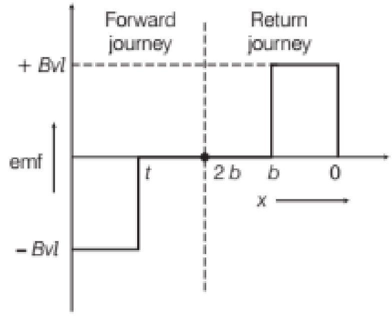
For $2b \ge x \ge b \Rightarrow e = 0$

Backward journey

For $b > x \ge 0 \Rightarrow e = \nu Bl$

For $2b \ge x \ge b$, $e = 0$

Variation of induced emf



29. Here, $A = 20 \text{ cm} \times 30 \text{ cm} = 6 \times 10^{-2} \text{ m}^2$

$B = 0.3$ T

Let θ be the angle made by the field B with the normal to the plane of the coil.

(i) Here, $\theta = 90° - 90° = 0°$

So, flux, $\phi = BA \cos\theta$

$\phi = 0.3 \times 6 \times 10^{-2} \times \cos 0°$

$= 1.8 \times 10^{-2}$ Wb

(ii) Here, $\theta = 90° - 30° = 60°$

$\phi = 0.3 \times 6 \times 10^{-2} \times \cos 60°$

$\phi = 0.9 \times 10^{-2}$ Wb

(iii) Here, $\theta = 90°$

$\phi = 0.3 \times 6 \times 10^{-2} \times \cos 90°$

$\phi = 0$

30. As we know that, $e = \frac{d\phi}{dt}$

As, $\phi = 5t^3 + 4t^2 + 2t$

So, $e = 15t^2 + 8t + 2$

For $t = 2$ s, $e = 15 \times 2^2 + 8 \times 2 + 2$

$= 60 + 16 + 2 = 78$ V

31. Here, radius, $r = 10 \text{ cm} = 10^{-1}$ m, $N = 500$ turns

Resistance, $R = 2\,\Omega$, $\theta_1 = 0°$, $\theta_2 = 180°$

$dt = 0.25$ s, $e = ?$, $I = ?$, $B = 3 \times 10^{-5}$ T,

$A = \pi r^2 = 3.14\,(10^{-1})^2 = 3.14 \times 10^{-2} \text{ m}^2$

$$\Rightarrow \quad e = -\frac{N\,(d\phi)}{dt} = -\frac{N\,(\phi_2 - \phi_1)}{dt}$$

$$= -\frac{NBA\,(\cos\theta_2 - \cos\theta_1)}{dt}$$

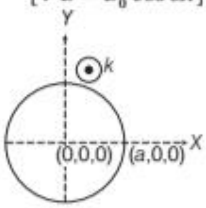
$$= -\frac{500 \times 3 \times 10^{-5} \times 3.14 \times 10^{-2}(\cos 180° - \cos 0°)}{0.25}$$

$$= \frac{2 \times 500 \times 3 \times 3.14 \times 10^{-7}}{0.25} = 3.8 \times 10^{-3}\text{ V}$$

and $I = \frac{e}{R} = \frac{3.8 \times 10^{-3}}{2} = 1.9 \times 10^{-3}$ A

32. At any instant, flux passing through the ring is given by $\phi = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta = BA$ $[\because \theta = 0]$

or $\phi = B_0(\pi a^2)\cos\omega t$ $[\because B = B_0\cos\omega t]$



By Faraday's law of electromagnetic induction,
the magnitude of induced emf is given by

$$e = \frac{d\phi}{dt} = B_0(\pi a^2)\,\omega\sin\omega t$$

This causes flow of induced current, which is given by

$$I = B_0(\pi a^2)\,\omega\sin\omega t / R$$

Now, finding the values of current at different instants. So, we have current at

$$t = \frac{\pi}{2\omega} \Rightarrow I = \frac{B_0(\pi a^2)\,\omega}{R} \text{ along } \hat{j}$$

Because $\sin\omega t = \sin\left(\omega\frac{\pi}{2\omega}\right) = \sin\frac{\pi}{2} = 1$

At $t = \frac{\pi}{\omega} \Rightarrow I = \frac{B_0(\pi a^2)\,\omega}{R} \times \sin\pi = 0$

Because $\sin\omega t = \sin\left(\omega\,\frac{\pi}{\omega}\right) = \sin\pi = 0$

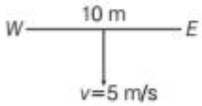
At $t = \frac{3\pi}{2\omega} \Rightarrow I = \frac{B_0(\pi a^2)\,\omega}{R}$ along $-\hat{j}$

Because $\sin\omega t = \sin\left(\omega\,\frac{3\pi}{2\omega}\right) = \sin\frac{3\pi}{2} = -1$

33. Refer to Example 7 on page 256, $B_H = 58 \times 10^{-3}$ T

The number of spokes is immaterial because the emf's across the spokes are in parallel.

34. Given, velocity of straight wire, $v = 5$ m/s



Horizontal component of the earth's magnetic field,

$$B = 0.30 \times 10^{-4}\text{ Wb/m}^2$$

Length of wire, $l = 10$ m

(i) The emf induced in the wire, $e = Blv\sin\theta$

Here, $\theta = 90°$

$\therefore \sin\theta = 1$ [$\because$ wire is falling at right angle to the earth's horizontal magnetic field component]

$$e = 0.30 \times 10^{-4} \times 10 \times 5$$
$$= 1.5 \times 10^{-3}\text{ V}$$

(ii) According to the Fleming's right hand rule, if the force is downward, then the direction of induced emf will be from West to East.

(iii) As the direction of induced emf is from West to East, the West end of the wire is at higher potential.

35. Given, speed of jet plane,

$$v = 1800\text{ km/h} = 1800 \times \frac{5}{18}$$
$$= 500\text{ m/s}$$

and l = distance between the ends of the wings = 25 m

The magnitude of magnetic field,

$$B = 5 \times 10^{-4}\text{ T}$$

Angle of dip, $\delta = 30°$

Use the formula of motional emf,

$$e = B_V vl \quad\text{or}\quad e = (B\sin\delta)\,vl$$

[$\because$ vertical component of the earth's magnetic field, $B_V = B\sin\delta$]

$$\Rightarrow e = 5 \times 10^{-4}\sin 30° \times 500 \times 25 = 3.1\text{ V}$$

Thus, the voltage difference developed between the ends is 3.1 V.

36. Refer to Example 7 on page 256, $e = 10$ V

37. Given, $B = 0.5$ T

$$l = 20\text{ cm} = 0.2\text{ m}$$
$$v = 10\text{ ms}^{-1}$$

emf induced $|e| = |Blv| = |-0.5 \times 0.2 \times 10| = 1$ V

Current in the arm, $I = \frac{e}{R} = \frac{1}{5} = 0.2$ A

38. Here, area, $A = 8 \times 2 = 16\text{ cm}^2 = 16 \times 10^{-4}\text{ m}^2$

$$B = 0.3\text{T},\ v = 1\text{ cm/s} = 10^{-2}\text{ m/s}$$

Induced emf, $e = ?$

(i) When velocity is normal to longer side,

$$l = 8\text{ cm} = 8 \times 10^{-2}\text{ m}$$
$$e = Blv = 0.3 \times 8 \times 10^{-2} \times 10^{-2}$$
$$= 2.4 \times 10^{-4}\text{ V}$$

Induced emf lasts till the loop comes out of field.

$$\text{Time, } t = \frac{\text{Distance covered by coil in uniform magnetic field}}{\text{Velocity of the coil}}$$

$$= \frac{2 \times 10^{-2}}{10^{-2}} \Rightarrow t = 2\text{ s}$$

(ii) When velocity is normal to shorter side,

$$l = 2\text{ cm} = 2 \times 10^{-2}\text{ m}$$

$$e = Blv = 0.3 \times 2 \times 10^{-2} \times 10^{-2}$$
$$= 0.6 \times 10^{-4}\text{ V}$$

$\therefore$ The induced voltage lasts for time,

$$t = \frac{\text{Distance covered by coil in uniform magnetic field}}{\text{Velocity of the coil}}$$

$$= \frac{8\times10^{-2}}{10^{-2}} = 8\text{ s}$$

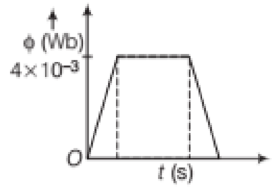
39. Given, $l = 20\text{cm} = 0.2\text{ m}$,

$B = 0.1\text{T},\ v = 10\text{ cms}^{-1} = 0.1\text{ms}^{-1}$

(i) Magnetic flux through loop $\phi = B\cdot A = Blx$

$\phi_{max} = 0.1\times0.2\times0.2$

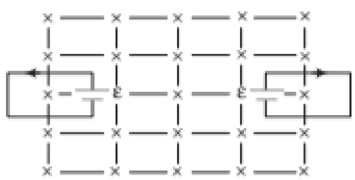
$= 0.004\text{ Wb} = 4\times10^{-3}\text{ Wb}$

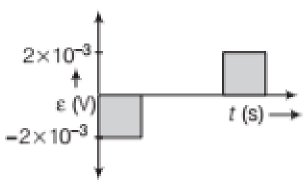


(ii) Induced emf, $\varepsilon = \frac{-d\phi}{dt} = -Blv$

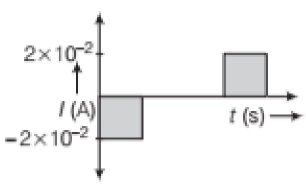
$\therefore |\varepsilon|_{max} = 0.1\times0.2\times0.1 = 0.002\text{ V}$

$= 2\times10^{-3}\text{V}$





(iii) Induced current, $I = \frac{|\varepsilon|}{R} = \frac{2\times10^{-3}}{0.1} = 2\times10^{-2}\text{ A}$



40. Given, side of loop, $a = 12\text{ cm}$

$\therefore$ Area of loop, $A = a^2 = (12)^2 = 144\text{ cm}^2 = 144\times10^{-4}\text{m}^2$

Velocity, $v = 8\text{ cm/s} = 8\times10^{-2}\text{ m/s}$ [X-axis]

Rate of change of magnetic field with distance,

$$\frac{dB}{dx} = 10^{-3}\text{ T/cm} \quad \text{[negative X-axis]}$$

Rate of change of magnetic field with time,

$$\frac{dB}{dt} = 10^{-3}\text{ T/s}$$

Resistance of the loop, $R = 4.5\text{ m}\Omega = 4.5\times10^{-3}\ \Omega$

Rate of change of magnetic flux with respect to time,

$$\frac{d\phi}{dt} = \frac{d(BA)}{dt} = \left(\frac{dB}{dt}\right)A \quad [\because \phi = BA]$$

$$= 10^{-3}\times144\times10^{-4}$$

$$= 1.44\times10^{-5}\text{ Wb/s}$$

Rate of change of magnetic flux due to the motion of loop,

$$\frac{d\phi}{dt} = \frac{dB}{dx}\cdot A\cdot\frac{dx}{dt} = 10^{-3}\times144\times10^{-4}\times8$$

$$\left[\because \frac{dx}{dt} = \text{velocity}\right]$$

$$= 11.52\times10^{-5}\text{ Wb/s}$$

Both of the effects cause a decrease in magnetic flux along the positive z-direction.

Total induced emf in the loop,

$$e = 1.44\times10^{-5} + 11.52\times10^{-5}$$

$$e = 12.96\times10^{-5}\text{ V}$$

Induced current in the loop $= \frac{e}{R} = \frac{12.96\times10^{-5}}{4.5\times10^{-3}}$

$$= 2.88\times10^{-2}\text{ A}$$

The direction of induced current is such as to increase the flux through the loop along positive z-direction, i.e. induced current will be anti-clockwise.

41. Given, radius of coil, $r = 8.0\text{cm} = 8\times10^{-2}\text{ m}$

$N = 20$ turns, $\omega = 50\text{ rad s}^{-1}$,

$B = 3\times10^{-2}\text{T},\ e_0 = ?,\ e_{av} = ?$

Resistance, $R = 10\ \Omega,\ P = ?$

As, $e_0 = NAB\omega = N(\pi r^2)B\omega \quad [\because A = \pi r^2]$

$$e_0 = 20\times\frac{22}{7}\times(8\times10^{-2})^2\times3\times10^{-2}\times50$$

$$e_0 = 0.603\text{ V}$$

Average value of emf induced over a full cycle,

$$e_{av} = 0$$

$$I_{max} = \frac{e_0}{R} = \frac{0.603}{10} = 0.0603\text{ A}$$

Average power dissipated,

$$P_{av} = \frac{e_0I_0}{2} = \frac{0.603\times0.0603}{2}$$

$$P_{av} = 0.018\text{ W}$$

The induced current causes a torque opposing the rotation of the coil. An external agent (rotor) must supply torque (and do work) to counter this torque in order to keep the coil rotating uniformly. Thus, the source of power dissipated as heat in the coil is the external rotor.

|TOPIC 2|

Self and Mutual Induction

INDUCTANCE

Flux linkage of a closely wound coil is directly proportional to the current I,

i.e. $\phi_B \propto I$

If the geometry of the coil does not varying with time, then

$$\frac{d\phi_B}{dt} \propto \frac{dI}{dt}$$

For a closely wound coil of N turns, the same magnetic flux is linked with all turns. The flux ϕ_B through the coil changes, each turn contributes to the induced emf. Therefore, flux linked with the coil (flux linkage) is equal to $N\phi_B$. In this case,

Total flux, $N\phi_B \propto I$

The constant of proportionality in this relation is called **inductance**. Therefore, inductance is basically a measure of the ratio of the flux to the current.

The SI unit of inductance is the tesla-square metre per ampere ($T\text{-}m^2/A$). We call this as henry (H), named in the honour of American physicist Joseph Henry, who discoveres the law of induction and a contemporary of Faraday and its dimensions are $[ML^2T^{-2}A^{-2}]$.

Thus, $1\ H = 1\ T\text{-}m^2/A$

Inductance is a scalar quantity which plays same role in an electrical circuit as played by inertia in mechanics. It depends only on the geometry of the coil and intrinsic material properties.

SELF-INDUCTANCE

It is the property of a coil by virtue of which, the coil opposes any change in the strength of current flowing through it by inducing an emf in itself. This induced emf is also called **back emf**. When the current in a coil is switched ON, the opposes the growth of the current and when the current is switched OFF, the self-induction opposes the decay of the current. So, self-induction is also called the **inertia of electricity**.

Coefficient of Self-Induction

Let us consider a coil of N turns carrying a current I. Let ϕ_B be the magnetic flux linked with each turn of the coil. Then, the number of flux linked through the coil will be $N\phi_B$.

If no magnetic materials (iron, etc.) are present near the coil, then the number of flux linkages with the coil is proportional to the current I, i.e.

$$N\phi_B \propto I \quad \text{or} \quad N\phi_B = LI$$

where, L is constant called the **coefficient of self-induction** or **self-inductance** of the coil.

By the above equation, we have

$$L = \frac{N\phi_B}{I} \qquad \text{...(i)}$$

If $I = 1$, then $L = N\phi_B$.

Hence, the coefficient of self-induction of a coil is numerically equal to the number of magnetic flux linkages with the coil when unit current is flowing through the coil.

If on changing the current through the coil, the back emf induced in the coil be e, then by Faraday's law, we have

$$e = -N\frac{\Delta\phi_B}{\Delta t} = -\frac{\Delta(N\phi_B)}{\Delta t}$$

where, $\Delta(N\phi_B)/\Delta t$ is the rate of change of magnetic flux (due to change of current) in the coil. But $N\phi_B = LI$.

$$\therefore \quad e = -\frac{\Delta(LI)}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

where, $\Delta I/\Delta t$ is the rate of change of current in the coil. The negative sign indicates that the induced emf e is always in such a direction that it opposes the change of current in the coil. From the above formula, we have

$$\boxed{L = -\frac{e}{\Delta I/\Delta t}} \qquad \text{...(ii)}$$

If $\Delta I/\Delta t = 1$, then $L = e$ (numerically).

Hence, the coefficient of self-induction of a coil is numerically equal to the emf induced in the coil when the rate of change of current in the coil is unity.

The SI unit of the coefficient of self-induction is henry (H) and its dimensions are $[ML^2T^{-2}A^{-2}]$. Thus, the self-inductance of a coil is 1 henry when an induced emf of 1volt is set up in the coil due to a current changing at the rate of 1 ampere per second in the coil, i.e.

$$1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ ampere/second}}$$

Thus, as before, $1\ H = 1\ Vs\ A^{-1} = 1 Wb\ A^{-1}$

But from Eq. (i), 1 henry = 1 weber/ampere.

The smaller units for L are millihenry (mH) and microhenry (μH).

$$1 \text{ mH} = 10^{-3} \text{ H}, \; 1 \, \mu\text{H} = 10^{-6} \text{ H}$$

Note When two coils of coefficient of self-induction L_1 and L_2 are

(i) connected in series, then $L = L_1 + L_2$.

(ii) connected in parallel, then $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$.

Self-Inductance of Long Solenoid

A long solenoid is one whose length is very large as compared to its area of cross-section. The magnetic field B at any point inside such a solenoid is practically constant and is given by

$$B = \frac{\mu_0 NI}{l} = \mu_0 \, nI \qquad \left[\because n = \frac{N}{l}\right] \quad \ldots \text{(i)}$$

where, μ_0 = magnetic permeability of free space,
N = total number of turns in the solenoid,
l = length of the solenoid
and n = number of turns per unit length.

$\therefore$ Magnetic flux through each turn of the solenoid,

$$\phi = B \times \text{area of the each turn}$$

$$\phi = \left(\mu_0 \frac{N}{l} I\right) A$$

where, A = area of each turn of the solenoid.

Total magnetic flux linked with the solenoid

= Flux through each turn × Total number of turns

$$N\phi = \mu_0 \frac{N}{l} IA \times N \quad \ldots \text{(ii)}$$

If L is coefficient of self-inductance of the solenoid, then

$$N\phi = LI \quad \ldots \text{(iii)}$$

From Eqs. (ii) and (iii), we get

$$LI = \mu_0 \frac{N}{l} IA \times N \text{ or } \boxed{L = \frac{\mu_0 N^2 A}{l}}$$

If core of any other magnetic material μ is placed, then

$$\mu = \mu_0 \, \mu_r \; [\mu_r = \text{relative magnetic permeability}]$$

$$\therefore \quad \boxed{L = \frac{\mu_0 \mu_r N^2 A}{l}}$$

Note This topic has been frequently asked in previous years 2015, 2014, 2013, 2012, 2011, 2010.

EXAMPLE |1| Current in a circuit falls steadily from 2.0 A to 0.0 A in 10 ms. If an average emf of 200 V is induced, calculate the self-inductance of the circuit.

Foreign 2011

Sol. Given, $\Delta I = -2 \text{ A}$, $\Delta t = 10 \times 10^{-3} \text{ s}$

$e = 200$ V, $L = ?$

$\because$ Induced emf, $e = -L \frac{\Delta I}{\Delta t} \Rightarrow 200 = -L \left(\frac{-2}{10 \times 10^{-3}}\right)$

$\Rightarrow \quad 200 = L \times 2 \times 10^2$

$\therefore$ Self-induction, $L = 1$ H

EXAMPLE |2| What is the self-inductance of a solenoid of length 40 cm, area of cross-section 20 cm² and total number of turns is 800?

Sol. Given, $l = 40 \text{ cm} = 0.4 \text{ m}$

$A = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$

$N = 800$, $L = ?$

$$\therefore \quad L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi \times 10^{-7} \times (800)^2 \times 20 \times 10^{-4}}{0.4}$$

$$= 4.02 \times 10^{-3} \text{ H}$$

EXAMPLE |3| In the circuit diagram shown in figure, $R = 10 \, \Omega$, $L = 5$ H, $E = 20$ V, $I = 2$ A. This current is decreasing at a rate of -1.0 A/s. Find V_{ab} at this instant.

Sol. Potential difference across inductor is given as

$$V_L = L \frac{dI}{dt} = (5)(-1.0) = -5 \text{ V}$$

Now, using Kirchhoff's second law, $V_a - IR - V_L - E = V_b$

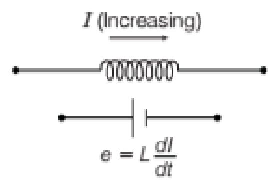
$$\therefore \quad V_{ab} = V_a - V_b = E + IR + V_L$$

$$= 20 + (2)(10) - 5$$

$$= 35 \text{ V}$$

Energy Stored in an Inductor

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field.



An increasing current in an inductor causes an emf between its terminals.

The work done per unit time is power,

$$\boxed{P = \frac{dW}{dt} = -eI = -LI \frac{dI}{dt}}$$

From $dW = -dU$

or $\frac{dW}{dt} = -\frac{dU}{dt}$, we have

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

or $dU = LI\,dI$

The total energy U supplied while the current increases from zero to a final value I is

$$U = L\int_0^I I dI = \frac{1}{2}LI^2$$

$$\therefore \quad \boxed{W = U = \frac{1}{2}LI^2}$$

Energy stored per unit volume(V) in magnetic field is known as **energy density.**

$$\therefore \quad \boxed{\text{Energy density} = \frac{U}{V} = \frac{1}{2}\frac{B^2}{\mu_0}}$$

Thus, if $I = 1$ A, then $2W = L$

Hence, the coefficient of self-inductance is equal to twice the work done in establishing a flow of one ampere current in the circuit.

EXAMPLE |4| Two coils having self-inductances, $L_1 = 5$mH and $L_2 = 1$mH. The current in the coil is increasing at same constant rate at a certain instant and the power supplied to the coils is also same. Find the ratio of

(i) induced voltages (ii) currents

(iii) energy stored in two coils at that instant.

Sol. Given, $L_1 = 5$mH and $L_2 = 1$mH

(i) As we know, induced voltage is given by $e = \frac{Ldi}{dt}$

$$\Rightarrow \quad \frac{e_1}{e_2} = \frac{L_1(dI/dt)}{L_2(dI/dt)} = \frac{L_1}{L_2} = \frac{5}{1} = 5:1 \quad \text{...(i)}$$

(ii) Power in the coil is given by

$$P = eI$$

Here, $P_1 = P_2 \Rightarrow e_1 I_1 = e_2 I_2 \Rightarrow \frac{I_1}{I_2} = \frac{e_2}{e_1}$

Using Eq. (i), we can write as, $\frac{i_1}{i_2} = \frac{1}{5} = 1:5$

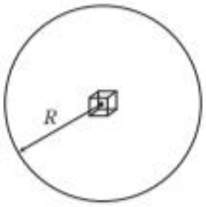
(iii) Energy stored in a coil is given by $U = \frac{1}{2}Li^2$

$$\therefore \quad \frac{U_1}{U_2} = \frac{(1/2)L_1I_1^2}{(1/2)L_2I_2^2} = \frac{L_1}{L_2}\left(\frac{e_2}{e_1}\right)^2$$

$$= \frac{5}{1}\left(\frac{1}{5}\right)^2 = 1:5$$

EXAMPLE |5| Suppose a cube of volume 2mm^3 is placed at the centre of a circular loop of radius 5cm carrying current 2A. Find the magnetic energy stored inside the cube.

Sol.



Magnetic field at the centre of the circular loop is given by $B = \frac{\mu_0 i}{2R}$.

We know, energy density, $\mu = \frac{B^2}{2\mu_0}$ and energy stored in the cube will be given by

$$U = \mu V_0 = \frac{B^2}{2\mu_0}V_0$$

where, V_0 is the volume of the cube.

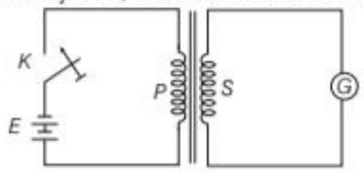
Substituting the values in the above equation,

$$= \frac{1}{2\mu_0}\times\left(\frac{\mu_0 I}{2R}\right)^2 V_0 = \frac{\mu_0 I^2 V_0}{8R^2}$$

$$= \frac{4\pi\times 10^{-7}\times 2^2\times 2\times 10^{-9}}{8\times(0.05)^2} = 16\pi\times 10^{-14}\ \text{J}$$

MUTUAL INDUCTANCE

The phenomenon according to which an opposing emf is produced in a coil (i.e. primary coil) as a result of change in current or magnetic flux linked with a neighbouring coil (i.e. secondary coil) is called **mutual induction**.



Mutual induction

Coefficient of Mutual Induction

Let a current of I_1 ampere flows in the primary coil (P). Due to this current, the magnetic flux linked with each turn of the secondary coil (S) be ϕ_2. If N_2 is the number of turns in the secondary coil, then the number of flux linkages in the coil will be $N_2\phi_2$. This number of flux linkages is proportional to the current I_1 flowing in the primary coil, i.e.

$$N_2\phi_2 \propto I_1 \quad \text{or} \quad N_2\phi_2 = MI_1$$

where, M is a constant called the **coefficient of mutual induction** or **mutual inductance** of the two coils. From the above equation, we have

$$M = \frac{N_2\phi_2}{I_1}$$

If $I_1 = 1$, then $M = N_2\phi_2$

Hence, the coefficient of mutual induction of two coils is equal to the number of magnetic flux linkages in one coil when a unit current flows in the other.

If on changing the current in the primary coil, the emf induced in the secondary coil is e_2, then according to Faraday's law, we have

$$e_2 = -N_2\frac{\Delta\phi_2}{\Delta t} = -\frac{\Delta(N_2\,\phi_2)}{\Delta t}$$

where, $\Delta\phi_2/\Delta t$ is the rate of change of magnetic flux in the secondary coil (due to change of current in the primary coil). But $N_2\phi_2 = MI_1$.

$$\therefore \quad e_2 = -\frac{\Delta(MI_1)}{\Delta t} = -M\frac{\Delta I_1}{\Delta t}$$

where, $\Delta I_1/\Delta t$ is the rate of change of current in the primary coil. The negative sign indicates that the direction of emf induced in the secondary coil is always such that it opposes any change in current in the primary coil.
From the above expression, we have

$$\text{Mutual inductance, } M = -\frac{e_2}{\Delta I_1/\Delta t}$$

If $\Delta I_1/\Delta t = 1$, then $M = e_2$ (numerically).

Hence, the coefficient of mutual induction of two coils is equal to the numerical value of the induced emf in one coil when the rate of change of current in other coil is unity.

The SI unit of the coefficient of mutual inductance is henry (H). Thus, the mutual inductance of two coils is 1 henry when an induced emf of 1 volt is set up in one of them due to a current changing at the rate of 1 ampere per second in the other.

i.e. $$1\text{ henry} = \frac{1\text{volt}}{1\text{ ampere/second}}$$

or $$1\text{H} = 1\text{V s A}^{-1} = 1\text{ Wb A}^{-1}$$

The coefficient of mutual induction or mutual inductance of two coils depend on

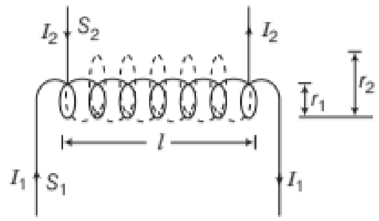
(i) geometry of two coils, i.e. size of coils, their shape, number of turns, nature of material on which two coils are wound.

(ii) distance between two coils.

(iii) relative placement of two coils (i.e. orientation of the two coils).

Mutual Inductance of Two Long Coaxial Solenoids

Consider two long solenoids S_1 and S_2 of same length l, such that solenoid S_2 surrounds solenoid S_1 completely.



Two long coaxial solenoids of same length l

Let n_1 be the number of turns per unit length of S_1, n_2 be the number of turns per unit length of S_2, I_1 be current passed through solenoid S_1 and ϕ_{21} be flux linked with S_2 due to current flowing through S_1.

$$\phi_{21} \propto I_1 \text{ or } \phi_{21} = M_{21}\,I_1$$

where, M_{21} is the coefficient of mutual induction of the two solenoids.

When current is passed through solenoid S_1, an emf is induced in solenoid S_2.Magnetic field produced inside solenoid S_1 on passing current through it.

$$B_1 = \mu_0 n_1 I_1$$

Magnetic flux linked with each turn of solenoid S_2 will be equal to B_1 times the area of cross-section of solenoid S_1.

Magnetic flux linked with each turn of the solenoid $S_2 = B_1 A$.

Therefore, total magnetic flux linked with the solenoid S_2 will be

$$\phi_{21} = B_1 A \times n_2 l$$
$$= \mu_0 n_1 I_1 \times A \times n_2 l$$
$$\phi_{21} = \mu_0 n_1 n_2 A I_1 l$$
$$M_{21} = \mu_0 n_1 n_2 Al \quad \text{...(i)}$$

Similarly, the mutual inductance between the two solenoids, when current is passed through solenoid S_2 and induced emf is produced in solenoid S_1 and is given by

$$M_{12} = \mu_0 n_1 n_2 Al$$
$$M_{12} = M_{21} = M \quad \text{(say)}$$

Hence, coefficient of mutual induction between two long solenoids,

$$M = \mu_0 n_1 n_2 Al$$

We can rewrite Eq. (i) as,

$$M = \mu_0\left(\frac{N_1}{l}\right)\left(\frac{N_2}{l}\right)\pi r_1^2 \times l$$

$$= \frac{\mu_0 N_1 N_2 A}{l}$$

If core of any other magnetic material μ is placed, then

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l}$$

Note This topic has been frequently asked in previous years 2015, 2013, 2012, 2011, 2010.

EXAMPLE |6| There are two coils, which have mutual inductance of 10 H. When the circuit is closed, current in the primary coil is raised to 3 A within a time range of 1 millisecond. Calculate the emf induced in secondary coil.

Sol. Given, mutual inductance, $M = 10$ H

Change in current, $dI = 3$ A

Change in time, $dt = 1$ millisecond $= 10^{-3}$ s

emf induced, $e = ?$

emf induced in secondary coil is given by $e = \frac{M\,dI}{dt}$

$\therefore$ emf induced $= \frac{10 \times 3}{10^{-3}} = 3 \times 10^4$ V

EXAMPLE |7| A 1 m long solenoid with diameter 2 cm and 2000 turns has a secondary coil of 1000 turns wound closely near its mid-point. What will be the mutual inductance between the two coils?

Sol. Given, $l = 1\text{m}, r = \frac{2}{2}\text{cm} = 1\text{ cm} = 10^{-2}\text{m}$

$$N_1 = 2000, N_2 = 1000, A = \pi r^2 = \pi (10^{-2})^2 \text{ m}^2$$

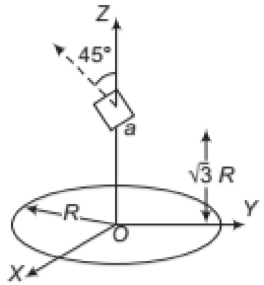
$$= \pi \times 10^{-4} \text{m}^2, M = ?$$

$\therefore$ Mutual inductance between the two coils is given by

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi \times 10^{-7} \times 2000 \times 1000 \times \pi \times 10^{-4}}{10^{-2}}$$

$$= 78.9 \times 10^{-3} \text{ H}$$

EXAMPLE |8| A circular wire loop of radius R is placed in the XY-plane centred at the origin O. A square loop of side a ($a << R$) having two turns is placed with its centre at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the Z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then find the value of p.



Sol. If I current flows through the circular loop, then magnetic field at the location of square loop is

$$B = \frac{\mu_0 I R^2}{2(R^2 + Z^2)^{3/2}}$$

Substituting the value of $Z = \sqrt{3}R$, we have

$$B = \frac{\mu_0 I}{16R}$$

Now, total flux through the square loop is

$$\phi_T = NBS \cos\theta$$

$$= (2)\left(\frac{\mu_0 I}{16R}\right) a^2 \cos 45°$$

Mutual inductance,

$$M = \frac{\phi_T}{I} = \frac{\mu_0 a^2}{2^{7/2}\,R}$$

$\therefore \quad p = 7$

AC GENERATOR

An AC generator produces electrical energy from mechanical work, just the opposite of what a motor does. In it, a shaft is rotated by some mechanical means, such as an engine or a turbine starts working and an emf is induced in the coil.

Principle

It is based on the phenomenon of electromagnetic induction which states that whenever magnetic flux linked with a conductor (or coil) changes, an emf is induced in the coil.

Main Parts of an AC Generator

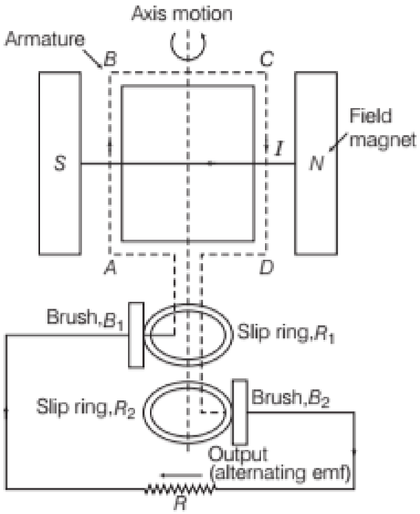
Main parts of an AC generator are shown in the figure and discussed as given below

Armature A rectangular coil $ABCD$ consisting of a large number of turns of copper wire wound over a soft iron core is called armature. The soft iron core is used to increase the magnetic flux.

Field magnets Two pole pieces of a strong electromagnet.

Slip rings The ends of the coil $ABCD$ are connected to two hollow metallic rings R_1 and R_2.

Brushes B_1 and B_2 are two flexible metal plates or carbon rods. They are fixed and are kept in slight contact with R_1 and R_2.



AC generator

Theory and Working

As the armature of coil is rotated in the uniform magnetic field, angle θ between the field and normal to the coil changes continuously.

Therefore, magnetic flux linked with the coil changes and an emf is induced in the coil. According to Fleming's right hand rule, current induced in AB is from A to B and it is from C to D in CD. In the external circuit, current flows from B_2 to B_1.

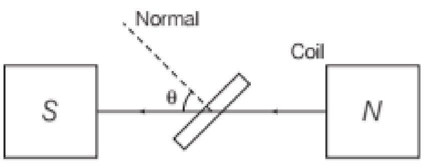
To calculate the magnitude of emf induced, suppose

A = area of each turn of the coil,

N = number of turns in the coil,

B = strength of magnetic field,

θ = angle which normal to the coil and $\mathbf{B}$ at any instant t.



Magnetic flux linked with the coil in this position,

$$\phi = N(\mathbf{B} \cdot \mathbf{A})$$
$$= NBA \cos\theta$$
$$= NBA \cos\omega t \qquad [\because \theta = \omega t]$$

where, ω is angular velocity of the coil and other symbols have usual meaning.

As, the coil rotates, angle θ changes. Therefore, magnetic flux ϕ linked with the coil changes and hence, an emf is induced in the coil. At this instant t, if e is the emf induced in the coil, then

$$e = -\frac{d\phi}{dt}$$
$$= -\frac{d}{dt}(NAB\cos\omega t)$$
$$= -NAB\frac{d}{dt}(\cos\omega t)$$
$$= -NAB(-\sin\omega t)\,\omega = NAB\,\omega\sin\omega t$$

where, $NBA\,\omega$ is the maximum value of the emf (also called peak value) which occurs when $\sin\omega t = \pm 1$.

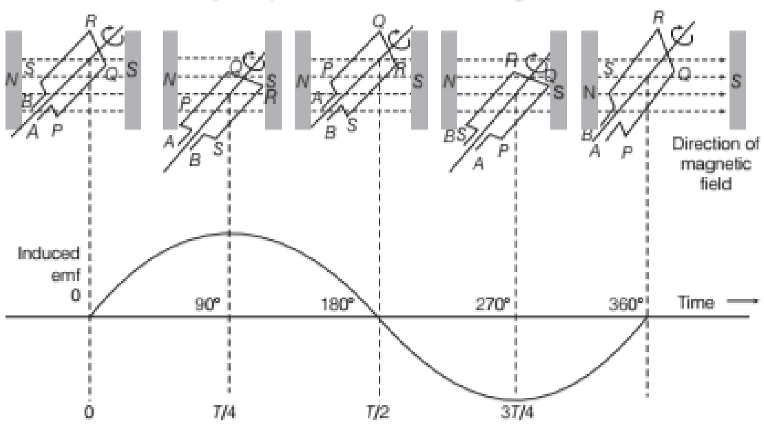
If $NBA\,\omega = e_0$, then $e = e_0 \sin\omega t$

Since, the value of the sine function varies between +1 and −1. So, the polarity of emf changes with time. The emf has its extreme value when $\theta = 90°$ or $\theta = 270°$ as the change of flux is greatest at these points. The direction of the induced emf (and hence current) changes periodically as shown in the figure given below. Therefore, the current is called **alternating current**.

Since, $\qquad \omega = 2\pi\nu$

$\therefore \qquad e = e_0 \sin 2\pi\nu t$

where, ν is the frequency of revolution of the generator's coil.



In commercial generators, the mechanical energy required for rotation of armature is provided by water falling from height, e.g. dams. These are called **hydroelectric generators**.

If the steam at high pressure is used to produce the rotation of armature, these are called **thermal generators**. Instead of coal, if a nuclear fuel is used, we get **nuclear power generators**.

Note In India, the frequency of generation of AC is 50 Hz.

EXAMPLE |9| An AC generator consists of a coil of 1000 turns and cross-sectional area of 100 cm^2, rotating at an angular speed of 100 rpm in a uniform magnetic field of 3.6×10^{-2} T. Calculate the maximum emf produced in the coil.

Sol. Given, $N = 1000, A = 100 \text{ cm}^2 = 10^{-2} \text{ m}^2$,

$$\nu = 100 \text{ rpm} = \frac{100}{60} \text{rps},$$

$$B = 3.6 \times 10^{-2} \text{T}, \; e_0 = ?$$

$\therefore$ Maximum emf produced in the coil is

$$e_0 = NBA\,\omega = NBA\,(2\pi\nu)$$

$$= 1000 \times 3.6 \times 10^{-2} \times 10^{-2} \times 2 \times \frac{22}{7} \times \frac{100}{60}$$

$$= 3.77 \text{ V}$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. The self-inductance of a coil is 2 mH. The rate of flow of current in it is 10^3 A/s. The induced electromotive force in the coil is

(a) 1 V (b) 2 V
(c) 3 V (d) 4 V

2. The self inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as

(a) l and A increase
(b) l decreases and A increases
(c) l increases and A decreases
(d) both l and A decrease

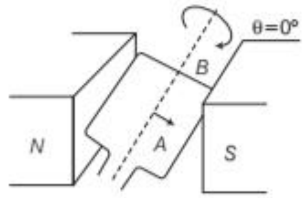
3. If a medium of relative permeability μ_r had been present instead of air, the mutual inductance would be

(a) $M = \mu_r \mu_0 n_1 n_2 \pi r_1 l$ (b) $M = \mu_0 n_1 n_2 \pi r_1^2 l$
(c) $M = \mu_r n_1 n_2 \pi r_1^2 l$ (d) $M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l$

4. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon

(a) the rates at which currents are changing in the two coils
(b) relative position and orientation of the two coils
(c) the materials of the wires of the coils
(d) the currents in the two coils

5. The effective area of the coil exposed to the magnetic field lines changes with time, the flux at any time is



(a) $\phi_B = BA \cot \omega t$
(b) $\phi_B = BA \cos \omega t$
(c) $\phi_B = BA \tan \omega t$
(d) $\phi_B = BA \sec \omega t$

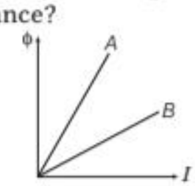
VERY SHORT ANSWER Type Questions

6. Can a straight wire act as an inductor?

7. Define the term self-inductance of a coil. Write its SI unit. **Delhi 2015**

8. How can the self-inductance of a given coil having N number of turns, area of cross-section A and length l be increased? **Foreign 2009; Delhi 2012**

9. A plot of magnetic flux (ϕ) *versus* current (I) is shown in the figure for two inductors A and B. Which of the two has larger value of self-inductance? **Delhi 2010**



10. Self-induction is called the inertia of electricity. Why?

11. Define mutual inductance. Write its SI unit. **Delhi 2016**

12. How does the mutual inductance of a pair of coils change when

(i) distance between the coils is increased and
(ii) number of turns in the coils is increased? **All India 2013**

SHORT ANSWER Type Questions

13. A source of emf e is used to establish a current I through a coil of self-inductance L. Show that the work done by the source to build up the current I is $\frac{1}{2}LI^2$. **Delhi 2010 C**

14. Define mutual inductance between two long coaxial solenoids. Find out the expression for the mutual inductance of inner solenoid of length l having the radius r_1 and the number of turns n_1 per unit length due to the second outer solenoid of same length and n_2 number of turns per unit length. **Delhi 2012**

15. Two concentric circular coils, one of radius r and the other of radius R are placed coaxially with their centres coinciding. For $R >>> r$, obtain an expression for the mutual inductance of the arrangement. **Delhi 2011**

16. A small square loop of wire of side l is placed inside a large square loop of wire of side $L (L >> l)$. The loops are coplanar and their centres coincide. Give the dependence of mutual inductance.

LONG ANSWER Type I Questions

17. (i) Define self-inductance. Write its SI units.
(ii) Derive the expression for self-inductance of a long solenoid of length l, cross-sectional area A having N number of turns. **Delhi 2009**

18. Starting from the expression for the energy, $W = \frac{1}{2}LI^2$, stored in a solenoid of self-inductance L to build up the current I, obtain the expression for the magnetic energy in terms of the magnetic field B, area A and length l of the solenoid having n number of turns per unit length. Hence, show that the energy density is given by $B^2/2\mu_0$. **Delhi 2013C**

19. The current through two inductors of self-inductance 12 mH and 30 mH is increasing with time at the same rate. Draw graphs showing the variation of the **All India 2017 C**
(i) emf induced with the rate of change of current in each inductor.
(ii) energy stored in each inductor with the current flowing through it.
Compare the energy stored in the coils, if the power dissipated in the coils is the same.

20. (i) Define the term 'self-inductance' and write its SI unit.
(ii) Obtain the expression for the mutual inductance of two long coaxial solenoids S_1 and S_2 wound one over the other, each of length L and radii r_1 and r_2; and n_1 and n_2 number of turns per unit length, when a current I is set up in the outer solenoid S_2. **Delhi 2017**

21. Define mutual inductance between a pair of coils. Derive an expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other. **All India 2017**

22. (i) Draw a schematic sketch of an AC generator describing its basic elements. State briefly its working principle. Show a plot of variation of
(a) magnetic flux and
(b) alternating emf *versus* time generated by a loop of wire rotating in a magnetic field.
(ii) Why is choke coil needed in the use of fluorescent tubes with AC mains? **Delhi 2014**

23. State the principle of an AC generator and explain its working with the help of a labelled diagram. Obtain the expression for the emf induced in a coil having N turns each of cross-sectional area A, rotating with a constant angular speed ω in a magnetic field (B), directed perpendicular to the axis of rotation. **CBSE 2018**

LONG ANSWER Type II Questions

24. (i) Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce a current which opposes the change of magnetic flux that produces it.
(ii) The current flowing through an inductor of self-inductance L is continuously increasing. Plot a graph showing the variation of
(a) magnetic flux *versus* current.
(b) induced emf *versus* dI/dt.
(c) magnetic potential energy stored *versus* the current. **Delhi 2014**

25. (i) Define mutual inductance and write its SI units.
(ii) Derive the expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other.
(iii) In an experiment, two coils C_1 and C_2 are placed close to each other. Find out the expression for the emf induced in the coil C_1 due to a change in the current through the coil C_2. **All India 2015**

26. (i) Explain the meaning of the term mutual inductance. Consider two concentric circular coils, one of the radius r_1 and the other of radius $r_2 (r_1 < r_2)$ placed coaxially with centres coinciding with each other. Obtain the expression for the mutual inductance of the arrangement.

(ii) A rectangular coil of area A, having number of turns N is rotated at f revolutions per second in a uniform magnetic field B, the field being perpendicular to the coil.

Prove that the maximum emf induced in the coil is $2\pi f NBA$. **All India 2016**

27. (i) Draw a labelled diagram of AC generator and state its working principle.

(ii) How is magnetic flux linked with the armature coil changes in a generator?.

(iii) Derive the expression for maximum value of the induced emf and state the induced emf and state the rule that gives the direction of the induced emf.

(iv) Show the variation of the emf generated *versus* time as the armature is rotated with respect to the direction of the magnetic fields. **Delhi 2014**

28. (i) State the principle on which AC generator works. Draw a labelled diagram and explain its working.

(ii) A conducting rod held horizontally along East-West direction is dropped from rest from a certain height near the Earth's surface. Why should there be an induced emf across the ends of the rod?

Draw a plot showing the instantaneous variation of emf as a function of time from the instant it begins to fall. **Foreign 2012**

29. State the working of AC generator with the help of a labelled diagram. The coil of an AC generator having N turns, each of area A, is rotated with a constant angular velocity ω. Deduce the expression for the alternating emf generated in the coil. What is the source of energy generation in this device? **Delhi 2010**

NUMERICAL PROBLEMS

30. A 200 turn coil of self-inductance 30 mH carries a current of 5 mA. Find the magnetic flux linked with each turn of the coil. **Delhi 2011**

31. Self-induction of an air core inductor increases from 0.01 mH to 10 mH on introducing an iron core into it. What is the relative permeability of the core used?

32. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V is induced, give an estimate of the self-inductance of the circuit. **NCERT**

33. If a rate of change of current of 4 As^{-1} induces an emf of 20 mV in a solenoid, what is the self-inductance of the solenoid? **Delhi 2010**

34. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside, normal to the axis of solenoid. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0.1 s, what is the induced voltage in the loop, while the current is changing? **NCERT**

35. A coil has a self-inductance of 10 mH. What is the maximum magnitude of the induced emf in the inductor, when a current $I = 0.1 \sin 200\, t$ ampere is sent through it.

36. A solenoid of radius 3 cm and length 1 m has 600 turns per metre. Calculate its self-inductance.

37. The current flowing in the two coils of self-inductance $L_1 = 16$ mH and $L_2 = 12$ mH are increasing at the same rate. If the power supplied to the two coils are equal, find the ratio of

(i) induced voltages (ii) the currents and

(iii) the energies stored in the coil at a given instant. **Foreign 2014**

38. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil? **Delhi 2016**

39. Two coils have mutual inductance of 1.5 H. If current in primary coil is raised to 5 A in one millisecond after closing the circuit, what is the emf induced in the secondary coil?

40. There are two coils A and B separated by some distance. If a current of 2 A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and a current of 1 A passes through B, what is the flux through A? **NCERT Exemplar**

41. The flux linked with a large circular coil of radius R is 0.5×10^{-3} Wb. When a current of 0.5 A flows through a small neighbouring coil of radius r, calculate the coefficient of mutual inductance for the given pair of coils. If the current through the small coil suddenly falls to zero, what would be its effect in the larger coil? **Delhi 2008**

42. An AC generator consists of coil of 100 turns and cross-sectional area of 3 m^2, rotating at a constant angular speed of 60 rad s^{-1} in a uniform magnetic field 0.04 T. The resistance of the coil is 500 Ω. Calculate (i) maximum current drawn from the generator and (ii) minimum power dissipation of the coil.

HINTS AND SOLUTIONS

1. (b) Given, coefficient of self-inductance,

$$L = 2 \times 10^{-3} \text{ H}$$

Rate of flow of current, $di/dt = 10^3$ A/s

Induced electromotore force,

$$|e| = \frac{Ldi}{dt} = 2 \times 10^{-3} \text{ V} = 2 \text{ V}$$

2. (b) The self-inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length, filled the inside of the solenoid with a material of relative permeability (*e.g.*, soft iron, which has a high value of relative permeability) is given by

$$L = \mu_r \mu_0 n^2 Al$$

where, $n = N/l$

3. (d) Air as the medium within the solenoids. Instead, if a medium of relative permeability μ_r had been present, the mutual inductance would be $M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l$.

It is also important to know that the mutual inductance of a pair of coils, solenoids etc., depends on their separation as well as their relative orientation.

4. (b) Mutual inductance of the pair of coils depends on distance between two coils and geometry of two coils.

5. (b) The effective area of the coil exposed to the magnetic field lines changes with time, the flux at any time t is

$$\phi_B = BA\cos\theta = BA\cos\omega t$$

6. A straight wire cannot act as an inductor as the magnetic flux linked with the wire of negligible area of cross-section is zero. The wire has to be in the form of a coil to serve as an inductor.

7. Self-inductance is the property of a coil by virtue of which, the coil opposes any change in the strength of current flowing through it by inducing an emf in itself. Its SI unit is henry (H).

8. The self-inductance can be increased with the help of electric fields. It does not depend on the current through circuit but depends upon the permeability of material from which the core is made up off.

9. Self-inductance of the inductor, $L = \phi / I$.

The slope of I-ϕ graph gives self-inductance of the coil. Inductor A have got greater slope than inductor B, therefore self-inductance of A is greater than self-inductance of B.

10. Self-induction of coil is the property by virtue of which it tends to maintain the magnetic flux linked with it and opposes any change in the flux by inducing current in it. This property of a coil is analogous to mechanical inertia, i.e. why self-induction is called the inertia of electricity.

11. The phenomenon according to which an opposing emf is produced in a coil as a result of change in current of magnetic flux linked with a neighbouring coil is called mutual induction.

Its SI unit is henry.

12. (i) As $\phi = MI$, with the increase in the distance between the coils, the magnetic flux linked with the secondary coil decreases and hence, the mutual inductance of the two coils will decrease with the increase of separation between them.

(ii) Mutual inductance of two coils can be found out by $M = \mu_0 N_1 N_2 Al$, i.e. $M \propto N_1 N_2$, so with the increase in number of turns, mutual inductance increases.

13. Refer to text on pages 269 and 270.

14. Refer to text on pages 271 and 272.

15. The magnetic field produced by current carrying large coil C_1 in the vicinity of small coil C_2 is given by

$$B_1 = \frac{\mu_0 I_1}{2R}$$

The magnetic flux linked with shorter coil C_2 is given by

$$\phi_2 = B_1 A_2 = \frac{\mu_0 I_1}{2R} \pi r^2$$

Mutual inductance, $M = \dfrac{\phi_2}{I_1} = \dfrac{\mu_0 \pi r^2}{2R}$ henry

16. Magnetic field produced by current carrying square loop of wire of side L is given by

$$B_1 = \frac{2\sqrt{2}\mu_0 I_1}{2L}$$

The magnetic flux linked with shorter square of side l is given by

$$\phi_2 = B_1 A_2 = \frac{2\sqrt{2}\mu_0 I_1}{2L} l^2$$

$\therefore$ Mutual inductance, $M = \dfrac{\phi_2}{I_1} = \dfrac{2\sqrt{2}\mu_0 l^2}{2L}$ henry

$$\Rightarrow \quad M \propto \frac{l^2}{L}$$

17. Refer to text on pages 268 and 269.

18. Energy stored in the magnetic field,

$$W = \frac{1}{2}LI^2 = \frac{1}{2}\cdot\frac{\mu_0 N^2 A}{l}\cdot\frac{B^2 l^2}{\mu_0^2 N^2}$$

$$= \frac{1}{2\mu_0}B^2 Al \qquad \left[\because L = \frac{\mu_0 N^2 A}{l},\ B = \frac{\mu_0 NI}{l}\right]$$

Energy density,

$$u_B = \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2\mu_0}B^2 AL}{AL} = \frac{B^2}{2\mu_0} \qquad [\because \text{volume} = Al]$$

19. Given, $L_1 = 12$ mH, $L_2 = 30$ mH

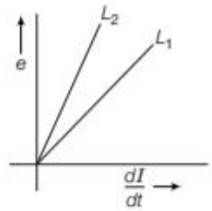
Also, $\frac{dI_1}{dt} = \frac{dI_2}{dt}$

(i) Induced emf in the inductors, $|e| = L\frac{dI}{dt}$

As, $\frac{dI_1}{dt} = \frac{dI_2}{dt}$

$\Rightarrow$ $e \propto L$

Thus, graph of e *versus* $\frac{dI}{dt}$ for two inductors is as shown in figure.



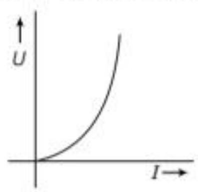
$\because$ $L_2 > L_1$

$\therefore$ For the given $\frac{dI}{dt}$, $e_2 > e_1$.

(ii) Energy stored in inductor, $U = \frac{1}{2}LI^2$

For a given L, $U \propto I^2$

Thus, U *versus* I graph is curved as shown in the figure.



Given, $P_1 = P_2$

$\Rightarrow e_1 I_1 = e_2 I_2 \ \Rightarrow \frac{I_1}{I_2} = \frac{e_2}{e_1}$

$$\therefore \quad \frac{U_1}{U_2} = \frac{\frac{1}{2}(L_1 I_1^2)}{\frac{1}{2}(L_2 I_2^2)} = \frac{L_1}{L_2}\left(\frac{I_1}{I_2}\right)^2 = \frac{L_1}{L_2}\left(\frac{e_2}{e_1}\right)^2$$

$$= \frac{12}{30}\left[\frac{L_2\,(dI/dt)}{L_1\,(dI/dt)}\right]^2$$

$$= \left(\frac{12}{30}\right)\times\left(\frac{30}{12}\right)^2 = \frac{30}{12} = \frac{5}{2}$$

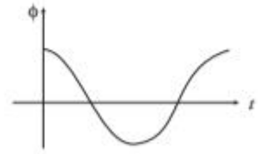
20. (i) Refer to text on page 268.

(ii) Refer to text on pages 271 and 272.

21. Refer to text on pages 271 and 272.

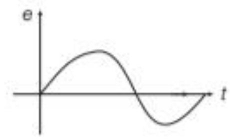
22. (i) Refer to text on pages 272 and 273.

(a) Variation of magnetic flux with time



$\phi = BNA\cos\omega t$

(b) Variation of alternating emf with time

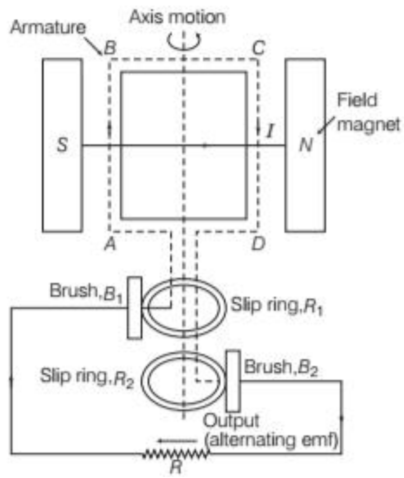


$e = BNA\,\omega\cos\omega t$

(ii) The choke coil is used to reduce the current. Therefore, it is required in the use of fluorescent tubes with AC mains.

23. **Principle**

An AC generator is based on the phenomenon of electromagnetic induction which states that whenever magnetic flux linked with a conductor (or coil) changes, an emf is induced in the coil.



Working

As the armature of coil is rotated in the uniform magnetic field, angle θ between the field and normal to the coil changes continuously.

Therefore, magnetic flux linked with the coil changes and an emf is induced in the coil. According to Fleming's right hand rule, current induced in AB is from A to B and it is from C to D in CD. In the external circuit, current flows from B_2 to B_1.

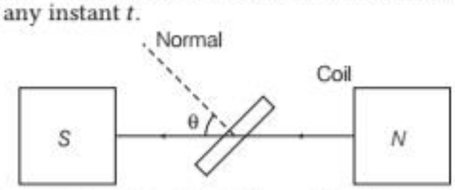
To calculate the magnitude of emf induced, suppose

$A =$ area of each turn of the coil,

$N =$ number of turns in the coil,

$B =$ strength of magnetic field

and $\theta =$ angle which normal to the coil makes with **B** at any instant t.



Magnetic flux linked with the coil in this position,

$$\phi = N(\mathbf{B}\cdot\mathbf{A})$$

or $$\phi = NBA\cos\theta$$

$$\phi = NBA\cos\omega t$$

where, ω is angular velocity of the coil and symbols have usual meaning.

As we know, due to the rotation of the coil, an emf is being induced.

Thus, at this instant t, if e is the emf induced in the coil, then

$$e = -\frac{d\phi}{dt}$$

$$e = -\frac{d}{dt}(NAB\cos\omega t)$$

$$e = -NAB\frac{d}{dt}(\cos\omega t)$$

or $$e = -NAB(-\sin\omega t) = NAB\sin\omega t$$

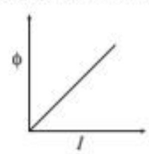
where, $NBA\omega$ is the maximum value of the emf (also called peak value) which occurs when $\sin\omega t = \pm 1$.

If $NBA\omega = e_0$, then $e = e_0 \sin\omega t$

24. (i) According to Lenz's law, the polarity of the induced emf is such that it opposes the change in magnetic flux responsible for its production.

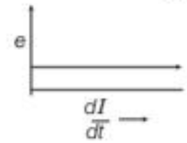
Refer to text on pages 253 and 254.

(ii) (a) **Magnetic flux *versus* current**



(b) **Induced emf *versus*** $\frac{dI}{dt}$

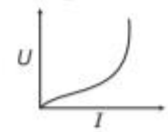
$$\Rightarrow \qquad e = -L\frac{dI}{dt}$$



$\frac{dI}{dt}$ is positive, and e is negative and constant.

(c) **Magnetic potential energy stored *versus* current**

$$U = \frac{1}{2}LI^2$$



$$\Rightarrow \qquad U \propto I^2$$

25. For parts (i) and (ii) refer to text on pages 270, 271 and 272.

(iii) Suppose that a current I is flowing through the coil C_2 at any instant. Flux linked with the coil C_1 is given by

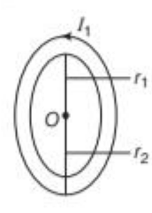
$$\phi \propto I \Rightarrow \phi = MI$$

where, M is the coefficient of mutual induction. If e is the induced emf produced in the coil C_1, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$$

26. (i) Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence, an emf will be induced in the neighbouring coil or circuit. This phenomenon is called 'mutual induction'.

According to question, let the current in big coil of radius r_2 be I_1, so magnetic field at point O due to this coil will be $\frac{\mu_0 I_1}{2r_2}$.



Change in magnetic flux in the coil of radius r_1 is

$$\phi = BA = \frac{\mu_0 I_1}{2r_2} \times \pi r_1^2$$

Mutual inductance,

$$M = \frac{\phi}{I_1} = \frac{\mu_0 I_1 \pi r_1^2}{2r_2 \times I_1} = \frac{\mu_0 \pi r_1^2}{2r_2}$$

This is the required expression.

(ii) According to question, if the coil rotates with an angular velocity of ω and N turns through an angle θ in time t, thus $\theta = \omega t$.

$\therefore \quad \phi = BA\cos\theta = BA\cos\omega t$

As the coil rotates, the magnetic flux linked with it changes. An induced emf is set up in the coil which is given by

$$e = \frac{-d\phi}{dt} = \frac{-d}{dt}(BA\cos\omega t)$$
$$= BA\omega\sin\omega t$$

For N number of turns, $e = NBA\omega\sin\omega t$

For maximum value of emf ωt must be equals to 90°.

So, maximum emf induced is = $NBA\omega$.

i.e. $\quad e = NBA2\pi f \quad [\because \omega = 2\pi f]$

27. Refer to text on pages 272 and 273.

28. (i) Refer to the text on pages 272 and 273.

(ii) As the earth's magnetic field lines are cut by the falling rod, the change in magnetic flux takes place. This change in flux induces an emf across the ends of the rod.

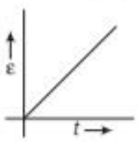
Since, the rod is falling under gravity.

$$v = gt \quad [\because u = 0]$$

Induced emf,

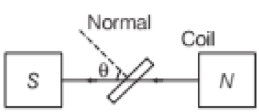
$$\varepsilon = Blv \Rightarrow \varepsilon = Blgt$$

$\therefore \quad \varepsilon \propto t$



29. Refer to text on pages 272 and 273.

Direction of induced emf can be determined using Fleming's right hand rule given below. If we stretch the thumb and the first two Fingers of our right hand in mutually perpendicular directions and if the forefinger points in the direction of the magnetic field, thumb in the direction of motion of the conductor, then the central finger points in the direction of current induced in the conductor.



30. Let ϕ be the magnetic flux linked with each of the N turns of the coil.

Then, $N\phi \propto I$

$\Rightarrow \quad N\phi = LI$ or $\phi = \dfrac{LI}{N}$

$\Rightarrow \quad \phi = \dfrac{30\times10^{-3}\times5\times10^{-3}}{200} = 7.5\times10^{-7}\,\text{Wb}$

31. Here, $L_0 = 0.01\,\text{mH} = 10^{-5}\,\text{H}$

$L = 10\,\text{mH} = 10^{-2}\,\text{H}$

$$\mu_r = ?,\ \mu_r = \frac{L}{L_0} = \frac{10^{-2}}{10^{-5}} = 10^3$$

or $\quad \mu_r = 1000$

32. 4 H; refer to Example 1 on page 269.

33. Here, $\dfrac{dI}{dt} = 4\,\text{As}^{-1}$

$\Rightarrow \quad |e| = 20\,\text{mV} \Rightarrow |e| = 20\times10^{-3}\,\text{V}$

$\Rightarrow \quad |e| = L\dfrac{dI}{dt}$

$\therefore \quad L = \dfrac{|e|}{dI/dt}$

$\Rightarrow \quad L = \dfrac{20\times10^{-3}}{4} = 5\times10^{-3}\,\text{H} = 5\,\text{mH}$

34. Here, number of turns per unit length,

$$n = \frac{N}{l} = 15\,\text{turns/cm} = 1500\,\text{turns/m}$$
$$A = 20\,\text{cm}^2 = 2\times10^{-4}\,\text{m}^2$$

$\therefore \quad \dfrac{dI}{dt} = \dfrac{4-2}{0.1}$ or $\dfrac{dI}{dt} = 20\,\text{As}^{-1}$

$\therefore \quad |e| = \dfrac{d\phi}{dt} = \dfrac{d}{dt}(BA) \quad \left[\because B = \dfrac{\mu_0 NI}{l}\right]$

$$= \frac{Ad}{dt}\left(\mu_0\frac{NI}{l}\right) = A\mu_0\left(\frac{N}{l}\right)\frac{dI}{dt}$$
$$= (2\times10^{-4})\times4\pi\times10^{-7}\times1500\times20\,\text{V}$$
$$= 7.5\times10^{-6}\,\text{V}$$

35. Here, $L = 10\,\text{mH} = 10^{-2}\,\text{H}$, $I = 0.1\sin 200t$

$\because \quad \dfrac{dI}{dt} = 0.1\cos 200t\times200 = 20\cos 200t$

$\Rightarrow \quad \left(\dfrac{dI}{dt}\right)_{\max} = 20\times1 = 20\,\text{As}^{-1}$

As, $\quad |e| = L\,(dI/dt) \Rightarrow e_{\max} = L\,(dI/dt)_{\max}$

$\Rightarrow \quad e_{\max} = 10^{-2}\times20 = 0.2\,\text{V}$

36. From the relation of self-inductance, we have

$$L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi\times10^{-7}\times(600)^2\times\pi\times(3\times10^{-2})^2}{1}$$
$$= 4\pi^2\times36\times10^4\times9\times10^{-11}$$
$$= 1.28\times10^{-3}\,\text{H}$$

37. Refer to the Example 4 on page 270.

(i) $\dfrac{e_1}{e_2} = \dfrac{4}{3}$ (ii) $\dfrac{I_1}{I_2} = \dfrac{3}{4}$ (iii) $\dfrac{E_1}{E_2} = \dfrac{3}{4}$

38. emf induced in the secondary coil is given by

$$e = \frac{-MdI}{dt}$$

$\Rightarrow \quad \dfrac{d\phi}{dt} = \dfrac{-MdI}{dt}$

$\Rightarrow \quad d\phi = -MdI$

or, $\quad d\phi = -1.5\times20 = -30\,\text{Wb}$

39. Given, $M = 1.5$ H, $\Delta i_1 = 5$A, $\Delta t = 10^{-3}$s

We know that, $M = -\dfrac{e_2}{\Delta i_1 / \Delta t}$

$$\Rightarrow \quad e_2 = M \times \frac{\Delta i_1}{\Delta t} = 1.5 \times \frac{5}{10^{-3}} = 7.5\ 10^3 \text{V}$$

40. Applying the mutual inductance of coil A with respect to coil B,

$$M_{21} = \frac{N_2 \phi_2}{I_1}$$

Therefore, we have

Mutual inductance $= \dfrac{10^{-2}}{2} = 5$ mH

Again, applying this formula for other case,

$$N_1 \phi_1 = M_{12} I_2 = 5 \text{ mH} \times 1 \text{ A} = 5 \text{ mWb}$$

41. Flux linked with larger coil of radius,

$$\phi = 0.5 \times 10^{-3} \text{ Wb and } I = 0.5 \text{ A}$$

Flows through neighbouring coil of radius r.

Total flux (ϕ) linked with one coil $= MI$

[$\because$ I current flows in neighbour coil]

$$\therefore \quad 0.5 \times 10^{-3} = M \times 0.5$$

$$M = 10^{-3} \text{ H}$$

Mutual inductance of two coils, $M = 10^{-3}$ H.

With the fall of current in small coil to zero, the magnetic flux linked with long coil decreases to zero quickly which in turn produces large induced emf in it.

42. Here, total number of turns,

$$N = 100, A = 3 \text{ m}^2, \omega = 60 \text{ rads}^{-1}, B = 0.04 \text{ T}$$

(i) Maximum emf produced in the coil,

$$e_0 = NBA\omega = 100 \times 0.04 \times 3 \times 60$$

$$e_0 = 720 \text{ V}$$

Since, resistance of the coil is 500 Ω, the maximum current drawn from the generator is

$$I_0 = \frac{e_0}{R} = \frac{720}{500} = 1.44 \text{ A}$$

(ii) Maximum power dissipation in the coil,

$$P = e_0 I_0 = 720 \times 1.44 = 1036.8 \text{ W}$$

SUMMARY

- **Magnetic Flux** The total number of magnetic field lines crossing through any surface normally when it is placed in a magnetic field is known as the magnetic flux through that surface.
- **Faraday's Law of EMI** Faraday gave two laws of EMI
  - (i) **First Law** An emf is induced in a circuit when the magnetic flux linked with circuit charges.
  - (ii) **Second Law** The magnitude of induced emf in a circuit is equal to the rate of change of magnetic flux through the circuit.
- **Induced emf and Current**

  Induced emf, $e = -N\frac{d\phi_B}{dt}$

  Induced current, $I = \frac{e}{R} = \frac{-N}{R}\frac{d\phi_B}{dt}$

  Here, R = resistance of the circuit and N = number of turns.
- **Lenz's Law** According to this law, the polarity of induced emf is such that it tends to produce a current which appose the change in magnetic flux produced it.
- **Fleming's Right Hand Rule** If we stretch the thumb, the forefinger, the central finger of our right hand in such a way that all three are mutually perpendicular to each other, then if the thumb represents the direction of force, forefinger represent the direction of magnetic field, then the central finger will represent the direction of induced current.
- **Induced Current in a Circuit** If induced current is produced in a coil rotated in a uniform magnetic field, then $I = I_0 \sin \omega t$.
- **Motional emf and Faraday's Law** If e is the induced emf, then according to Faraday's law,

$$e = (-d\phi/dt) = -Blv$$

- **Energy Consideration** Power required to move a conductor in a uniform magnetic field perpendicular is, $P = \frac{B^2 l^2 v^2}{R}$.

  Here, R = resistance of the circuit through which current is flowing.

  B = a uniform magnetic field, l = length of the conductor, v = speed.
- **Eddy Currents** The current induced in the bulk of conductors, when the magnetic flux linked with the condutor charges are known as eddy currents.
- **Undesirable Effects of Eddy Currents** Eddy Currents cause unnecessary heating and wastage of power.

  The heat produced by eddy currents may even damage the insulation of coils.
- **Inductance** It is the ratio of the flux to the current. It depends on the geometry of the coil and intrinsic material properties.
- **Self-inductance** It in the property of a coil by virtue of which it opposes any changes in the strength of current flowing through it by inducing an emf in itself.
- When current in a coil changes, it induces a back emf in the same coil. The self-induced emf is given by, $e = -L\frac{dI}{dt}$, where L is the self-inductance of the coil.
- If coil of N turns and area A is rotated with ν revolutions per second in a uniform magnetic field B, then the induced emf is $e = e_0 \sin \omega t = NAB\omega \sin \omega t = NBA(2\pi\nu)\sin(2\pi\nu)t$.
- **Self-inductance of a Long Solenoid** Self-inductance of a long solenoid is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

- **Mutual Inductance** The phenomenon according to which an opposing emf is produced as a result of change in current or magnetic flux linked with a neighbouring coil.
- **Mutual Inductance of Two Long Coaxial Solenoids** Mutual inductance of two long coaxial solenoids is given by

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

- **AC Generator** A generator produced electrical energy from mechanical work just opposite of what a motor does.

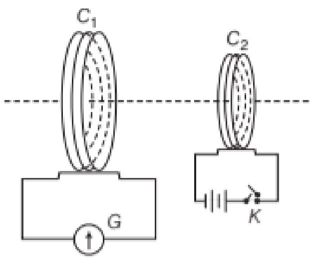
CHAPTER PRACTICE

OBJECTIVE Type Questions

1. A square of side L metres lies in the xy-plane in a region, where the magnetic field is given by $\mathbf{B} = B_0(2\hat{i} + 3\hat{j} + 4\hat{k})$ T, where B_0 is constant. The magnitude of flux passing through the square is **NCERT Exemplar**

(a) $2B_0L^2$ Wb (b) $3B_0L^2$ Wb
(c) $4B_0L^2$ Wb (d) $\sqrt{29}B_0L^2$ Wb

2. What will happen with the galvanometer when the tapping key K is pressed?



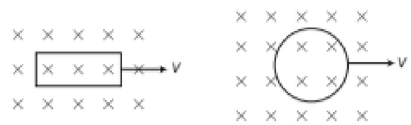
(a) A momentary deflection
(b) A long time deflection
(c) No deflection
(d) None of the above

3. The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit, is statement of
(a) Fleming's right hand rule
(b) Fleming's left hand rule
(c) Felming's third law
(d) Faraday's law of electromagnetic induction

4. The direction of induced current is decided by
(a) Lenz's law
(b) Fleming's left hand rule
(c) Biot-Savart's law
(d) Ampere's law

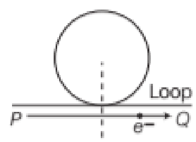
5. A 50 turns circular coil has a radius of 3 cm, it is kept in a magnetic field acting normal to the area of the coil. The magnetic field B increased from 0.10 T to 0.35 T in 2 ms^{-1}. The average induced emf in the coil is
(a) 1.77 V (b) 17.7 V (c) 177 V (d) 0.177 V

6. A rectangular loop and a circular loop are moving out of a uniform magnetic field region in the given figure to a field free region with a constant velocity v. In which loop do you expect the induced emf to be constant during the passage out of the field region?



(a) Rectangular loop (b) Circular loop
(c) Both (a) and (b) (d) Neither (a) nor (b)

7. An electron moves along the line PQ which lies in the same plane as a circular loop of conducting wire as shown in figure. What will be the direction of the induced current in the loop?



(a) Anti-clockwise
(b) Clockwise
(c) Alternating
(d) Non-current will be induced

8. A constant current is flowing through a solenoid. An iron rod is inserted in the solenoid along its axis. Which of the following quantities will not increase? **CBSE 2021 (Term-I)**
(a) The magnetic field at the centre
(b) The magnetic flux linked with the solenoid
(c) The rate of heating
(d) The self-inductance of the solenoid

9. A coil of area 100 cm^2 is kept at an angle of 30° with a magnetic field of 10^{-1}T. The magnetic field is reduced to zero in 10^{-4}s. The induced emf in the coil is **CBSE 2021 (Term-I)**
(a) $5\sqrt{3}$ V (b) $50\sqrt{3}$ V
(c) 5.0 V (d) 50.0 V

10. The self-induced emf in a coil of 0.4 H self-inductance when current in it is changing at the rate of 50 As^{-1}, is

(a) 8×10^{-4} V (b) 8×10^{-3} V

(c) 20 V (d) 500 V

ASSERTION AND REASON

Directions (Q. Nos. 11-16) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

11. Assertion Faraday's law are consequence of conservation of energy.

Reason In purely resistive AC circuit, the current lags behind the emf in phase.

12. Assertion Lenz's law violates the principle of conservation of energy.

Reason Induced emf always opposes the change in magnetic flux responsible for its production.

13. Assertion In equation $\mathbf{F} = q(\mathbf{E} + v \times \mathbf{B})$ when $v = 0$, any force on the charge must arise from the electric field term E alone.

Reason To explain, the existence of induced emf or induced current in static conductor kept in time-varying magnetic field, we must assume that a time-varying magnetic field generates an electric field.

14. Assertion Eddy currents are undesirable.

Reason Eddy currents heat up the core and dissipate electrical energy in the form of heat.

15. Assertion Eddy current is produced in any metallic conductor when magnetic flux is changed around it.

Reason Electric potential determines the flow of charge.

16. Assertion If the inner solenoid was much shorter than (and placed well inside) the outer solenoid, then we could still have calculated the flux linkage $N_1\phi_1$.

Reason The inner solenoid is effectively immersed in a uniform magnetic field due to the outer solenoid.

CASE BASED QUESTIONS

Directions (Q.Nos. 17-18) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

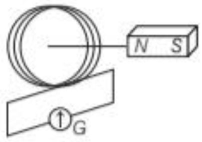
17. Faraday's Laws

According the Faraday's first law, whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in it. Induced current is determined by the rate at which the magnetic flux changes.

Mathematically, the magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through the circuit.

Induced emf $\propto$ Rate of change of magnetic flux

(i) On the basis of Faraday's law, current in the coil is larger



(a) when the magnet is pushed towards the coil faster

(b) when the magnet is pulled away the coil faster

(c) Both (a) and (b)

(d) Neither (a) nor (b)

(ii) The flux linked with a circuit is given by $\phi = t^3 + 3t - 7$. The graph between time (X-axis) and induced emf (Y-axis) will be a

(a) straight line through the origin

(b) straight line with positive intercept

(c) straight line with negative intercept

(d) parabola not through the origin

(iii) Wire loop is rotated in a magnetic field. The frequency of change of direction of the induced emf is

(a) once per revolution

(b) twice per revolution

(c) four times per revolution

(d) six times per revolution

(iv) The instantaneous magnetic flux linked with a coil is given by $\phi = (5t^3 - 100t + 300)$ Wb. The emf induced in the coil at time $t = 2$ s is

(a) – 40 V (b) 40 V

(c) 140 V (d) 300 V

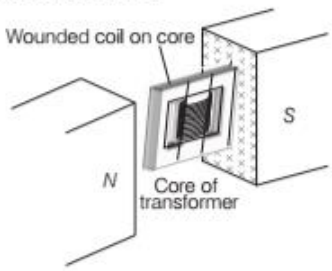
(v) A copper disc of radius 0.1 m is rotated about its centre with 20 rev/s in a uniform magnetic field of 0.1 T with its plane perpendicular to the field. The emf induced across the radius of the disc is

(a) $\frac{\pi}{20}$ V (b) $\frac{\pi}{10}$ V

(c) 20π mV (d) None of these

18. Eddy Current

Coil is wound over metallic core is helpful in reducing eddy currents in the metallic cores of transformers, electric motors, induction furnaces and other such devices (as shown below). Eddy current are undesirable since they heat up the core and dissipate electrical energy in the form of heat. These currents are minimised by using laminations of metal to make a metal core.



(i) How are eddy currents minimised to make a metal core of transformer on which coils are wound?

(a) By using laminations of metal
(b) By using solid metallic core
(c) Both (a) and (b)
(d) Neither (a) nor (b)

(ii) The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the

(a) keep on sliding
(b) keep on rotating
(c) cut across the induced eddy currents
(d) Both (a) and (b)

(iii) Induction furnace is used to produce

(a) low temperature to melt the metal
(b) high temperature to melt the metal
(c) constant low temperature 20°C
(d) high pressure

(iv) Induction furnace can be utilised to prepare

(a) alloys, by melting the constituent metals
(b) metal, by mixing electrons, protons, neutrons
(c) Both (a) and (b)
(d) Neither (a) nor (b)

(v) When a high frequency alternating current is passed through a coil which surrounds the metal to be melted. Then,

(a) the metal freezes
(b) coil rotates with frequency ω
(c) the metal melts
(d) None of the above

VERY SHORT ANSWER Type Questions

19. On what factors does the magnitude of induced emf in a coil depend?

20. If a coil is removed from a magnetic field
(i) slowly and
(ii) rapidly, then in which case, more work will be done?

21. Why is a core of transformer laminated?

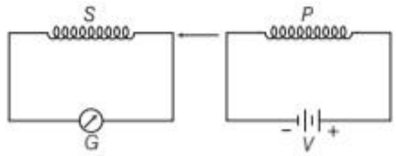
22. Give any two useful applications of eddy currents.

23. How can self-inductance of a given coil having N number of turns be changed, if N is doubled keeping other factors constant?

24. If two coils are very tightly wound over one another, will their mutual inductance increase or decrease as compared to the case when the coils are placed some distance apart?

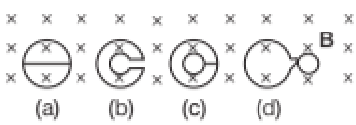
SHORT ANSWER Type Questions

25. (i) When primary coil P is moved towards secondary coil S (as shown in the figure below), the galvanometer shows momentary deflection. What can be done to have larger deflection in the galvanometer with the same battery?

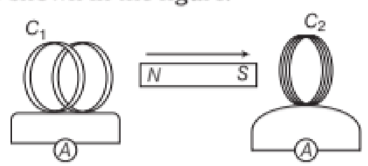


(ii) State the related law. **Delhi 2010**

26. Four shapes made of wires are situated in a magnetic field B, perpendicular to the plane of the paper, directed downwards. If B starts reducing, what will be the directions of the induced currents in these shapes?

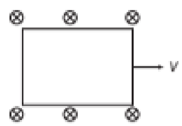


27. A magnet is quickly moved in the direction indicated by an arrow between two coils C_1 and C_2 as shown in the figure.



What will be the direction of induced current in each coil as seen from the magnet? Justify your answer.

28. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular to the plane of the loop exists everywhere. What will be the current induced?



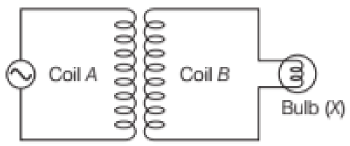
29. Two concentric circular coils C_1 and C_2, radius r_1 and r_2 ($r_1 << r_2$) respectively are kept coaxially. If current is passed through C_2, then find an expression for mutual inductance between the two coils.

LONG ANSWER Type I Questions

30. The figure given below shows an arrangement by which current flows through the bulb X connected with coil B, when AC is passed through coil A.

(i) Name the phenomenon involved.

(ii) If a copper sheet is inserted in the gap between the coils, explain how the brightness of the bulb would change?



31. State the law that gives the polarity of the induced emf. Give its illustration.

LONG ANSWER Type II Questions

32. State and explain Faraday's law of electromagnetic induction.
A cylindrical bar magnet is placed along the axis of a circular coil. Will there be a current induced in the coil, if magnet is rotated about its axis?

33. State Lenz's law. Give one example to illustrate this law. Lenz's law is a consequence of principle of energy conservation. Justify this statement.

34. How is the mutual inductance of a pair of coils affected, when

(i) separation between the coils is increased?

(ii) the number of turns of each coil is increased?

(iii) a thin iron sheet is placed between the two coils, other factors remaining the same? Explain your answer in each case.

NUMERICAL PROBLEMS

35. A jet plane is travelling towards West at a speed of 450 m/s. If the horizontal component of the earth's magnetic field at that place is 4×10^{-4} T and dip angle is 30°, calculate the emf induced between the ends of wings having a span of 30 m.

36. A long solenoid of 10 turns/cm has a small loop of area 1 cm^2 placed inside with the normal of the loop parallel to the axis. Calculate the voltage across the small loop, if the current in the solenoid is changed from 1 A to 2 A in 0.1 s, during the duration of this charge.

37. A solenoid of length 80 cm, area of cross-section 1 m^2 with 20 turns per cm completely surrounds another coaxial solenoid of the same length, area of cross-section 25 cm^2 with 20 turns per cm. Calculate the mutual inductance of the system.

ANSWERS

1. (c)	2. (a)	3. (d)	4. (a)	5. (b)
6. (a)	7. (d)	8. (c)	9. (a)	10. (c)

11. (c) According to Faraday's law of the conservation mechanical energy into electrical energy is in accordance with the law of conservation of energy. It is also clearly known that in pure resistance, the emf is in phase with the current.

12. (d) Lenz's law is based on conservation of energy and induced emf always opposes the cause of it, i.e. change in magnetic flux.

13. (a) Increasing **B** causes induced current I due to induced electric field E along I.

Note This induced electric field is non-conservative, makes closed loop electric field lines.

So, time varying magnetic field generates electric field.

14. (a)

15. (*b*)

16. (*a*) If the inner solenoid was much shorter than (and placed well inside) the outer solenoid, then we could still have calculated the flux linkage $N_1\phi_1$ because the inner solenoid is effectively immersed in a uniform magnetic field due to the outer solenoid.

17. (i) (*c*) Current will be larger, when the magnet is pushed faster towards the coil, also current is large when magnet is pulled faster away but now it is in opposite direction.

(ii) (*d*) $\phi = t^3 + 3t - 7$

$\therefore$ Induced emf, $e = -\frac{d\phi}{dt} = -(3t^2 + 3) = -3t^2 - 3$

At $t = 0; e = -3\text{ V}$

Therefore, shape of graph will be a parabola not through origin. $(\because e \propto t^2)$

(iii) (*b*) If a wire loop is rotated in a magnetic field, the frequency of change in the direction of the induced emf is twice per revolution.

(iv) (*b*) Given, $\phi = (5t^3 - 100t + 300)$, $t = 2\text{ s}$

Induced electromotive force,

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(5t^3 - 100t + 300)$$

$$e = -5 \times 3t^2 + 100 = -5 \times 3(2)^2 + 100$$

$$= -5 \times 12 + 100 = -60 + 100 = 40\text{ V}$$

(v) (*c*) From Faraday's law of electromagnetic induction,

$$e = -\frac{d\phi}{dt} = -BAN \quad (\because dt = 1\text{s})$$

Given, $B = 0.1$ T, $N = 20$, $A = \pi r^2 = \pi(0.1)^2$

$\therefore \quad e = 0.1 \times 20 \times \pi(0.1)^2 = 20\pi$ mV

18. (i) (*a*) Eddy current are minimised by using laminations of metal to make a metal core.

(ii) (*c*) The laminations are separated by an insulating material like lacquer. The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths. This arrangement reduces the strength of the eddy current. Since, the dissipation of the strength of electric current, heat loss is substantially reduced.

(iii) (*b*) Induction furnace can be used to produce high temperatures and can be utilised to prepare alloys, by melting the constituent metals. A high frequency alternating current is passed through a coil which surrounds the metals to be melted. The eddy currents generated in the metals produce high temperatures sufficient to melt it.

(iv) (*a*) Since, induction furnaces uses the concept of eddy currents. Thus they are used to prepare alloys, by melting the constituent metals.

(v) (*c*) Eddy current generated in the metal produce high temperature sufficient to melt it.

19. Number of turns in coils, and rate of change of magnetic flux.

20. More work will be done in (ii) Case.

21. To prevent it from eddy current being produced in the core.

22. In electric power meters and in induction furnace.

23. Doubled.

24. Increases

25. Refer to text on page 252.

26. Refer to text on page 253.

27. Refer to Q. 10 on page 259.

28. Current induced is zero.

29. Refer to Q. 15 on page 275.

30. Refer to Q. 11 on page 259.

31. **Lenz's law** Refer to text on pages 253 and 254.

32. Refer to text on pages 252 and 253.

33. Refer to text on pages 253 and 254.

34. Refer to text on page 271.
Also, refer Q. 12 on page 274.

35. $e = 3$ kV.

Refer to Q. 35 on page 262.

36. Voltage $= 12.57 \times 10^{-7}$ V

Refer to Q. 34 on page 276.

37. Mutual inductance = 3.92 H

Refer to Example 7 on page 272.